

Raw Material Inventory Control on Probabilistic Demand and Lead Time Using Continuous Review System

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Abstract

Excess inventory will lead to high inventory costs caused, among others, by damage and possible loss of stored goods embedded capital; on the other hand, a list is needed to ensure fulfillment of needs and reduce demand fluctuations and supply fluctuations to avoid loss of profit opportunities. This study aims to manage the uncertainty of usage and lead time in a wooden toy company, which can result in an excess or shortage of raw materials in the inventory system. It is necessary to control inventory to ensure optimal inventory; that is, needs can be met with minimal inventory costs and adequate service levels. The result, optimal condition is achieved if the company orders wood when the amount of inventory is $(r) = 24,422$ units with an order quantity of $(Q) = 10,010$ units each time you place an order, with a total inventory cost in 2023 of IDR 362,156,168, this policy will result in a service level $(\eta) = 95.14\%$.

Keywords: continuous review system, inventory control, lead time, probabilistic demand, raw material

INTRODUCTION

Inventory is idle resources awaiting further processing that can impact the operations, marketing, and finance functions. Inventory is caused by unsynchronized demand with supply, time used to process raw materials, and economic reasons. The unsynchronization is caused by the uncertainty of demand, storage, and time to process raw materials. In the supply chain system, inventory can be analogized as a water tank, delay not only from the direction of demand but also from the direction of supply, such as uncertainty in delivery time, number of shipments, and product prices, which can encourage companies to have excess inventory in reserve. In an industry, the financial effect of inventory on the supply chain can reach 25% or more of the total value of assets owned.

The primary function of inventory management is related to planning needs, both in terms of quantity and time, when an item is needed with adequate quality at the optimum cost level based on consideration of usage factors and lead time for item procurement. Regarding the material requirements in the future, inventory problems can be deterministic or probabilistic, and ordering times can be static or dynamic. Excess inventory will lead to high inventory costs caused, among others, by damage and possible loss of stored goods embedded capital; on the other hand, a list is needed to ensure fulfillment of needs and reduce demand fluctuations and supply fluctuations to avoid loss of profit opportunities. Good inventory management is not only able to reduce inventory costs but also able to increase service levels to customers.

One of the highly responsive inventory systems is the continuous inventory system. This system allows organizations to gain better visibility into their inventory, manage inventory more efficiently, and make better decisions based on accurate and real-time information. These are some of the key advantages offered by continuous inventory systems. Several ongoing inventory control studies based on several conditions have been developed, including the Meta-Heuristic Algorithms approach (Fattahi et al., 2015), Periodic Review (Park et al., 2014), and uncertain receiving quantity (Priyan & Uthayakumar, 2015), backorders and lost sales (Indrawati et al., 2018; Gozali et al., 2013; Estellés et al., 2012; Bijvank and Vis 2011;), back orders and investment (Uthayakumar & Parvathi, 2009), random demand (Braglia et al., 2019; Agrawal et al., 2013), permissible delay in payments (Skouri et al., 2011; Teng et al., 2011), and trade credits (Wu et al., 2018).

To overcome the complexity of managing inventory management in monitoring and determining the timing of orders in a continuous inventory system, the Continuous Review System can be used in deterministic and

probabilistic situations. In connection with the uncertainty of usage and lead time in wooden toy companies, which can result in excess or shortage of raw materials in the inventory system, it is necessary to control inventory aimed at ensuring optimal inventory, that is, needs can be met with minimal inventory costs and adequate service levels.

BASIC THEORY

A probabilistic inventory model is an approach that utilizes the concept of probability to anticipate and overcome uncertainty in the demand and supply of goods. This model allows companies to understand the probability distribution of demand and utilize this information for more intelligent decision-making. In a probabilistic inventory model, various aspects, such as variations in customer demand, supplier lead times, and other levels of uncertainty, are systematically considered. Thus, companies can avoid problems such as overstock or understock, which can negatively affect profits and customer service.

Two probabilistic inventory control methods are known: the Q and the P methods. 1. Q and P method with a back order, an inventory control method that allows for a shortage of inventory (backorder) when demand exceeds available inventory. 2. Q and P method with lost sales is an inventory control method that does not allow backorders or inventory shortages, but customers will look for goods elsewhere. In the Q method with probabilistic demand and lead time with lost sales, the values of Q^* and r^* can be determined as follows:

1. Determine the value of $Q1^*$ as follows:

$$Q1^* = \sqrt{\frac{2A}{Dh}} \quad (2.1)$$

2. Determine the values of α and $r1^*$ using the following formulas:

$$\alpha = \frac{h Q1^*}{Cu (D)+h (Q1^*)} \quad (2.2)$$

$$r1^* = D L + Z\alpha Sdl \quad (2.3)$$

3. Determine the values of $Q2^*$:

$$Q2^* = \sqrt{\frac{2D(A + Cu \int_r^\infty (x-r)f(x)dx)}{h}}, \text{ with} \quad (2.4)$$

$$\int_r^\infty (x-r)f(x)dx = Sdl [f(Z_\alpha) - Z_\alpha \Psi(Z_\alpha)]$$

4. Determine the values of α and $r2^*$ using the following formulas:

$$\alpha = \frac{h Q2^*}{Cu (D)+h (Q2^*)} \quad (2.5)$$

$$r2^* = D L + Z\alpha Sdl \quad (2.6)$$

5. Compare the $r1^*$ and $r2^*$; if the value of $r2^*$ is relatively the same as $r1^*$, the iteration is complete and obtained $r^* = r2^*$ and $Q1^* = Q2^*$. Otherwise, return to step three by replacing the $r1^* = r2^*$ and $Q1^* = Q2^*$.

6. Calculate the total inventory cost $(Ot)_0$ using the following formula:

7. Ot = purchase cost + order cost + storage cost + out-of-stock cost:

$$Ot = D p + \frac{AD}{Q} + h \left(\frac{Q}{2} + r - D_L + \int_r^\infty (x-r)f(x)dx \right) + \frac{Cu D}{Q} \int_r^\infty (x-r)f(x)dx \quad (2.7)$$

The level of service (η) that describes the probability of demand fulfillment is:

$$\eta = (1 - \int_r^\infty (x-r)f(x)dx/DL) \times 100\%$$

Notation:

Ot	= Total inventory cost
D	= Expected quantity of material required
p	= price per unit of material
A	= Cost each time place an order
Q	= Number of units ordered each time place an order
h	= Storage cost per unit per period
r	= Amount of inventory at the time the order is placed
D_L	= Expected demand during lead time
x	= Random variable of demand for goods during the lead period
$f(x)$	= Probability density function of random variable x

- C_u = Inventory shortage Cost per unit of material
- η = Service Level
- α = Probability of inventory shortage
- Z _{α} = Z value of the standard normal distribution for level α
- L = Lead time
- Sdl = Standard deviation of probabilistic demand and lead time

RESEARCH METHODS

Research Design

Raw materials are one of the basic needs in a manufacturing industry. Effective and efficient inventory management is needed to provide raw materials in the right amount and time at a minimum cost. This research aims at the time and amount of ordering raw materials to obtain minimal costs with a relatively high service level.

Research Object

The research was conducted in the PPIC section, with the object of study being raw materials that are categorized as critical for the toy industry made from wood.

Research Flow

The research flow is as follows:

- a. Identification of raw material requirements
- b. Identify and calculate the amount of inventory, including purchase costs, messages, storage costs, and inventory shortage costs.
- c. Identify the amount of lead time
- d. Forecast raw material requirements
- e. Perform normality test
- f. Determine the optimal conditions: when (when) to place an order and the number of raw materials to be ordered each time to place an order, the maximum expected inventory, and service level.
- g. Discussion and Closing.

DATA ANALYSIS AND DISCUSSION

Raw Material Requirements

The following data is obtained based on data on raw material requirements that are considered critical in the section on PPIC (which is degenerate):

Table 1. Raw material requirements for 2022

Month	Demand (unit)	Month	Demand (unit)	Month	Demand (unit)
1	95.106	5	127.840	9	166.865
2	114.018	6	148.995	10	133.438
3	133.248	7	119.018	11	160.175
4	106.385	8	143.025	12	186.866
Total					1.634.979

Inventory Costs

Inventory costs are costs that occur as a result of having inventory, which include:

- a. Purchase cost describes the purchase price of raw materials to suppliers, IDR 256,000 / unit, and there is no discount for any purchase.
- b. Ordering costs describe the costs the company bears every time it orders, including telephone charges, inspection fees, transportation, and administration amounting to IDR 500,000.
- c. Storage costs, including capital costs, warehouse labor costs, electricity costs, and warehouse administration per year of 12.5% of the price of raw materials.
- d. Inventory shortage costs are incurred due to inventory not being at a particular time and not meeting demand, so the raw material procurement department must make emergency procurement/ordering. The amount of the cost of inventory shortage is IDR 120.00 / unit.

Lead Time

The lead time required is relatively variable, with an average time range of 2 - 4 days. The company places orders once per month, with the lead time in 2022 being as follows:

Table 2 Predicted raw material requirements in 2022

Lead Time (days)	Frequency
2	4
3	5
4	3
Lead time average= 2,917 days	

ANALYSIS AND DISCUSSION

Based on the data above, it can be determined:

Forecasting Raw Material Requirements

The optimal forecast is obtained by the Holt-Winters Additive Algorithm method at the seasonal cycle length or (c) = 3; smoothing constant alpha = 0.93; smoothing constant beta = 0.32; smoothing constant gamma = 0 with MAPE = 2.186% (highly accurate category), obtained graphs and predictions of raw material needs in 2023 as follows:

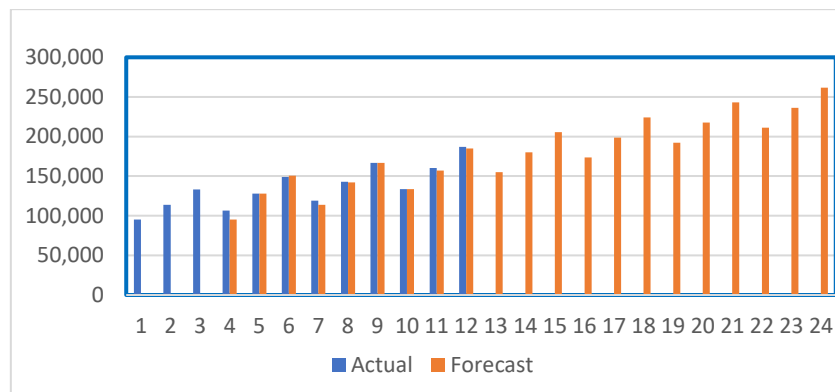


Figure 1. Raw material requirements

The results of forecasting raw material requirements are as follows:

Table 3. Raw Material Requirements

Month	Demand (unit)	Month	Demand (unit)	Month	Demand (unit)
1	154.833	5	198.750	9	242.986
2	179.996	6	224.232	10	211.095
3	205.478	7	192.341	11	236.258
4	173.587	8	217.504	12	261.739

Normality Test

Based on the normality test of raw material requirements in 2023 using Kolmogorov Smirnov, the following results are obtained:

Table 4. Normal test result

No	X	D = F(Z) - F(X)	No	X	D = F(Z) - F(X)
1	154.833	0,042	7	211.095	0,046
2	173.587	0,037	8	217.504	0,048
3	179.996	0,071	9	224.232	0,051
4	192.341	0,031	10	236.258	0,014
5	198.750	0,038	11	242.986	0,046

6	205.478	0,036	12	261.739	0,041
Dmax					0,071

$D_{cal} = 0.071 < 0.375$, it can be concluded that the data is normally distributed.

Inventory Control

Based on the prediction of raw material demand in 2023, inventory costs, and lead time in 2023, assuming the same magnitude as in 2022, the amount is obtained:

- Purchase cost (P) = IDR 256,000, - / unit.
- Raw material requirements/year (D) = 2,498,799 units
- Average lead time = 2.917 days = 2.917/300 years
- Order cost, - / once a message (A) = IDR 500,000, -
- Cost to store/unit/year (h) = 12.5% × IDR 256,000,- = IDR 32,000,-
- Inventory shortage cost (/unit Cu) = IDR 120,-
- Var(p) = 945,952,783, Var(l) = 699×10-8, Sdl = 3,082.16

Total Inventory Cost

$$Ot = D p + \frac{AD}{Q} + h \left(\frac{Q}{2} + r - D_L + \int_r^{\infty} (x - r)f(x)dx \right) + \frac{Cu D}{Q} \int_r^{\infty} (x - r)f(x)dx$$

Because there is no discount for any purchase, in the following analysis, the purchase cost is not included in the model.

Iteration 1:

- $$Q1^* = \sqrt{\frac{2AD}{h}}$$

$$= ((2 \times 500.000 \times 2.498.799) / (32.000))^{1/2}$$

$$= 8.836,71$$
- $$\alpha = \frac{h Q1^*}{Cu (D)+h (Q1^*)}$$

$$= 32.000 \times 8.836,71 / (120 \times 2.498.799 + 32.000 \times 8.836,71) = 0,48530$$

$$Z\alpha = 0,03675$$

$$r1^* = D L + Z\alpha Sdl$$

$$= 2.498.799 (2,917/300) + 0,03675 (3.082,16)$$

$$= 24.407,15$$
- $$Q2^* = \sqrt{\frac{2D(A + Cu \int_r^{\infty} (x-r)f(x)dx)}{h}}$$

with

$$\int_r^{\infty} (x - r)f(x)dx = Sdl [f(Z\alpha) - Z\alpha\Psi(Z\alpha)]$$

$$Z\alpha = 0,03675 ; f(Z\alpha) = 0,39873 ; \Psi(Z\alpha) = 0,38084$$

$$\int_r^{\infty} (x - r)f(x)dx = 3.082,16 (0,39873 - 0,03675 \times 0,38084) = 1185,63$$

$$Q2^* = \sqrt{\frac{2 \times 2.498.799 (500.000 + 120 \times 1.185,63)}{32.000}} = 10.015,36$$
- $$\alpha = \frac{h Q2^*}{Cu (D)+h (Q2^*)}$$

$$= 32.000 \times 10.015,36 / (120 \times 2.498.799 + 32.000 \times 10.015,36)$$

$$= 0,51663$$

$$Z\alpha = 0,04170$$

$$r2^* = D L + Z\alpha Sdl$$

$$= 2.498.799 (2,917/300) + 0,04170 (3.082,16)$$

$$= 24.422,41$$

Since the value of $r2^* = 24,422.41$ is relatively different from $r1^* = 24,407.15$, then go back to step c by changing the value of $r1^* = r2^*$ and $Q1^* = Q2^*$.

Iteration 1:

$$\begin{aligned} \text{c) } r1^* &= 24.422,41; Q1^* = 10.015,36 \\ \alpha &= 0,51663; Z\alpha = 0,04170; f(Z\alpha) = 0,39860; \Psi(Z\alpha) = 0,37844 \\ \int_r^\infty (x-r)f(x)dx &= Sdl [f(Z\alpha) - Z\alpha\Psi(Z\alpha)] \\ \int_r^\infty (x-r)f(x)dx &= 3.082,16 (0,39860 - 0,04170 \times 0,37844) = 1.179,89 \\ Q2^* &= \sqrt{\frac{2D(A + Cu \int_r^\infty (x-r)f(x)dx)}{h}} \\ Q2^* &= \sqrt{\frac{2 \times 2.498.799 (500.000 + 120 \times 1.179,89)}{32.000}} = 10.009,99 \end{aligned}$$

$$\begin{aligned} \text{d) } \alpha &= \frac{h Q2^*}{Cu(D) + h(Q2^*)} \\ &= \frac{32.000 \times 10.009,99}{120 \times 2.498.799 + 32.000 \times 10.009,99} \\ &= 0,51650 \\ Z\alpha &= 0,04137 \\ r2^* &= D L + Z\alpha Sdl \\ &= 2.498.799 (2,917/300) + 0,04137 (3.082,16) \\ &= 24.421,38 \approx 24.422 \end{aligned}$$

$$\begin{aligned} \text{e) } \text{Karena nilai } r2^* &= 24.421,38 \text{ relatif sama dengan } r1^* = 24.422,41, \text{ maka iterasi selesai dan diperoleh} \\ r^* &= 24.421,38 \text{ dan } Q^* = 10.009,99 \approx 10.010 \\ Ot &= \frac{AD}{Q^*} + h \left(\frac{Q^*}{2} + r^* - D_L + \int_r^\infty (x-r)f(x)dx \right) + \frac{Cu D}{Q} \int_r^\infty (x-r)f(x)dx \\ &= \frac{500.000 \times 2.498.799}{10.010} + 32.000 \left(\frac{10.010}{2} + 24.421,38 - 2.498.799 (2,917/300) + \right. \\ &\quad \left. 1.179,89 \right) + \frac{120 \times 2.498.799}{10.010} 1.179,89 = 362.156.168 \end{aligned}$$

$$\begin{aligned} \eta &= (1 - \int_r^\infty (x-r)f(x)dx/DL) \times 100\% = (1 - 1.179,89/(2.498.799 \times 2,917/300)) \times 100\% \\ &= 95,14\% \end{aligned}$$

This value indicates that the optimal condition is achieved if the company orders 10,010 units when the warehouse's wood inventory is 24,422 units.

CONCLUSION

Based on the results of the above analysis, it can be concluded that:

The optimal condition is achieved if the company orders wood when the amount of inventory is $(r) = 24,422$ units with an order quantity of $(Q) = 10,010$ units each time you place an order, with a total inventory cost in 2023 of IDR 362,156,168, - this policy will result in a service level $(\eta) = 95.14\%$.

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