(IJID) International Journal on Informatics for Development,
Vol. 8, No. 1, 2019, Pp. 28-34

Nonlinear Controller Design via Inferenced-Augmentation of Equivalent Linearized System

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***Abstract*—Output compensation of nonlinear system response is studied in this work with an inference-based design method used to control the nonlinear dynamics in the presence of non-linear effects. This design which is driven by practical considerations is informed by the comparisons made between nonlinear models and their linearized derivatives. While a detailed mathematical route has not been followed here, the results seem to show the workability of the proposed method and at its core, the design is driven by its intuitiveness. The proposed technique utilizes as primary parameter for comparison, the steady state error term which is derived from the behavior of the linear dynamics. Possible application areas for this design is in reduced energy control of nonlinear systems. The method was tested in simulation on a generic nonlinear system and a cart-driven inverted pendulum benchmark system.**

Keywords-nonlinear dynamics; linearization; steady state; control systems; hierarchical control

I. INTRODUCTION

Although many nonlinear system controller design techniques exist [1], [2], there has and still exists in the nonlinear research community the continuous search into synthesizing adequate or diluted solutions (not necessarily optimal) for any given nonlinear system [3]. Having a system model as the starting point for most of the known investigations made, while bearing in mind the inherent nonlinear behavior of natural system dynamics [4], [5]. Designing a controller that is both time, energy and computationally efficient remains a quality most sought after by control engineers in controlled systems that are inherently nonlinear.

While rigorous mathematical involvement is needed in many of the treatment for nonlinear controller design [6], the proposed method does not suggest the overhaul of more established methods, especially as it concerns stability analysis and system inversion [7]. Rather, the paper aims to explore another path to quick controller design and implementation which is seen to be a viable alternative if adopted in certain types of systems requiring a "not too" strict mathematical involvement in the final implementation of a controller. Such controller design for nonlinear systems can be applied directly to systems where the control margin is acceptable within a specified error tolerance.

This work used the system steady state error as a design metric to measure the controller performance against a nominal controller formulation [8]. The approach bears some similarities to observers only in the following sense [9], [10]; that the nonlinear system response is forced to track the nominal system output response equivalent, and in the utilization of tunable gains that can force the desired output to converge faster or slower as needed [11]. However, no claim is being made here concerning the proposed method as having the mathematical rigor that accompanies acceptable observer implementation [12], [13]. The results show that better performance response was obtained with the new controller algorithm not only in the originally intended steady state error reduction but also in stabilization and perfect tracking of the system output response.

The rest of the paper is arranged as follows; Section 2 introduces the working models for a general nonlinear systems and an inverted pendulum on a cart benchmark system, Section 3 addresses simulation and analysis, Section 4 discusses results and finally Section 5 summarizes the main findings and concludes.

II. EQUATIONS

The equations of motion for the two systems selected are presented in the following order; firstly, the nonlinear model representations are given. Next, the equivalent linearized models are presented for consideration. Both systems are considered as uncertain considering the unknown parts of the individual dynamics of the system and inherent modeling errors.

A. Generalized Nonlinear System

Given the nonlinear second order system defined by (1);

$$\begin{aligned} \dot{x}_1 &= -2x_1 + ax_2 + \sin(x_1) \\ \dot{x}_2 &= -x_2\cos(x_1) + u\cos(2x_1) \end{aligned} \tag{1}$$

where the state are x1 & x2 respectively and the input is u.

Further, the vector fields f(x) and g(x), which represents the state and input components of the system are at least C^1 and defined as (2) and (3) respectively;

$$f(x) = \begin{cases} -2x_1 + ax_2 + \sin(x_1) & f_1(x) \\ -x_2\cos(x_1) & f_2(x) \end{cases} \tag{2}$$

and similarly the g(x) component of the system is given by;

$$g(x) = \begin{cases} 0 & g_1(x) \\ \cos(2x_1) & g_2(x) \end{cases} \tag{3}$$

from which an equivalent Jacobian linearized system is obtained at the equilibrium points (x1e; x2e) = (0; 0) and (u1e; u2e) = (0; 0) as;

$$\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \end{pmatrix} = \begin{pmatrix} -1 & a \\ 0 & -1 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \begin{pmatrix} 0 \\ 1 \end{pmatrix} u \tag{4}$$

from which, the linearized system is given by

$$\begin{aligned} \dot{x}_1 &= -x_1 + ax_2 \\ \dot{x}_2 &= -x_2 + u \end{aligned} \tag{5}$$

Let the output be taken on x1 or y = x1 respectively i.e. c= [1 0], the linearized model has the transfer function given by (6),

$$G(s) = \frac{Y(s)}{U(s)} \equiv \frac{a}{s^2 + 2s + 1} \tag{6}$$

B. Cart-Driven Inverted Pendulum (CIP) Benchmark System

The nonlinear CIP system was adopted [14] and is presented here

$$\begin{aligned} M\ddot{x} + ml\ddot{\theta}\cos\theta - ml\dot{\theta}^2\sin\theta &= u \\ J\ddot{\theta} + mgl\sin\theta &= -ml\ddot{x}\cos\theta \end{aligned} \tag{7}$$

M=m+M represents the combined mass of pendulum bob and cart, while $J=I+ml^2$ or in some other variants, [15] $\mathbf{J} = \frac{2}{3}\mathbf{m}l^2$ represents combined rotational inertia of pendulum bob about the axis of rotation on the cart.

Rearranging both equations such that $\ddot{x}$ and $\ddot{\theta}$ are the dependent terms of the equations results in

$$\ddot{x} = \frac{1}{MJ}\left(\frac{1}{J}m^2l^2g\cos\theta\sin\theta + ml\dot{\theta}^2 + u\right)$$
$$\ddot{\theta} = \frac{1}{JM}\left(-m\lg\sin\theta - \frac{1}{M}m^2l^2\dot{\theta}^2\cos\theta\sin\theta - \frac{1}{M}ml\cos\theta u\right) \quad (8)$$

Utilizing the following state assignments $x_i = [\mathrm{x}, \dot{\mathrm{x}}, \theta, \dot{\theta}]$, the state space form of (8) is

$$\dot{x}_1 = x_2$$
$$\dot{x}_2 = \frac{1}{MJ}\left(\frac{m^2l^2g}{J}\cos x_3\sin x_3 + mlx_4^2 + u\right) \quad (9)$$
$$\dot{x}_3 = x_4$$
$$\dot{x}_4 = \frac{1}{JM}\left(-ml\,g\sin x_3 - \frac{m^2l^2}{M}x_4^2\cos x_3\sin x_3 - \frac{ml}{M}\cos x_3 u\right)$$

Equation (7) linearized yields

$$M\ddot{x} - ml\ddot{\theta} = u$$
$$J\ddot{\theta} - m\lg\theta(1+\delta) = \mathrm{ml}\,\ddot{x} \quad (10)$$

Repeating the same sequence of operations as with the nonlinear system yields the linear state space form;

$$\dot{x}_1 = x_2$$
$$\dot{x}_2 = \frac{1}{MJ}\left(\frac{m^2l^2g}{J}x_3(1+\delta) + u\right) \quad (11)$$
$$\dot{x}_3 = x_4$$
$$\dot{x}_4 = \frac{1}{JM}\left(-ml\,g\,x_3(1+\delta) + \frac{ml}{M}u\right)$$

Below is summarized the various parameters used in the equations

TABLE I. DERIVED PARAMETERS FOR CIP

		Nonlinear CIP	Linear CIP
1	MJ	$M - \frac{m^2l^2}{J}\cos^2\theta$	$M - \frac{m^2l^2}{J}$
2	JM	$J - \frac{m^2l^2}{M}\cos^2\theta$	$J - \frac{1}{M}m^2l^2$

Definitions for J and M were given previously

III. SIMULATION & ANALYSIS OF THE SYSTEM IN (1)

Simulation studies were carried out on Matlab/Simulink platform. The transfer function of the system in equation 8 is stable, because the characteristic polynomial is strictly proper with all poles real and negative. For the simulation, it suffices that the value of the variable a, be positive such that $0 < a < 1$ or more explicitly written the term a which represents unknown parameter influences on the system can be made to obey $|a| \leq 1$.

The unit step response in open loop test is given in Fig. 1.

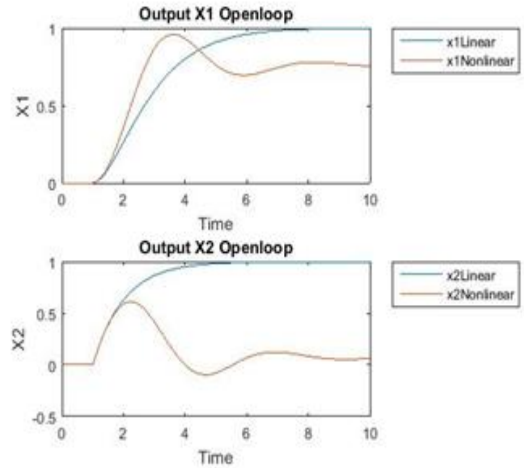

Figure 1. Closed Loop System (5 units Step Response

Fig 1 shows the results from direct application of a unit step alimenting the linear (4), and nonlinear (1) models in open loop or without feedback.

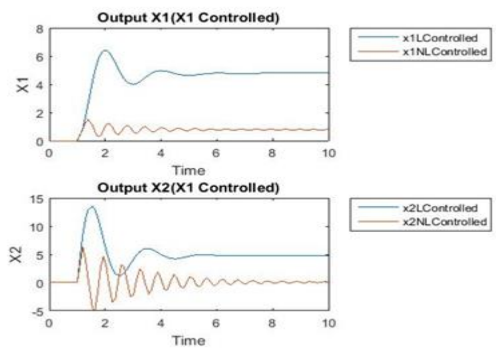

Figure 2. Open Loop System (Unit Step Response)

A proportional-integral-derivative (PID) controller from Matlab is then utilized with tuned PID gains given as Kp = 2:033; Ki =1:420; Kd = 0:652 on the linearized system (4). With the same controller utilized on the nonlinear model (as-is before linearization), the resulting output is shown in the plot of Fig. 2. The reference or set point was chosen as 5 units.

Fig. 2, shows this output response of the system when the state

x1 is fed-back into the system, with a reference signal of 5 unit step input.

Standard quantifiers or response metrics are: Rise time, Overshoot, Settling time, Steady state error, which are used in characterization of linear system response to commanded signal. For the subsequent analysis, the selected parameter for the inference tuning is the steady state error. This discrepancy parameter is quantified and introduced into the nonlinear model in an augmented control scheme.

A. *Discussion of Preliminary Simulation Results: Without Output Augmentation*

From results for the closed loop behavior, considering both the linearized and original nonlinear system we see the clearly discernible steady state error on the output signal for the step response. To make this comparison clear, both results have been superimposed onto the same graph as shown in Fig. 2. Through computation of output values in both linear and nonlinear case or by using a dynamic device like an observer, the state error metric is easily obtained. This error is then used to compensate for the large undershoot in response of nonlinear output. The method for the augmented regulated output is described in section IV.

IV. Controller Inference Design Technique

The design of the inference controller introduced in this work, makes use of the knowledge of state error. The error computation proceeds as follows;

$$\begin{aligned} Aug_{err} &= output_{lin} - output_{nlin} \\ G_{nlin} &= output_{nlin} + Aug_{err} \end{aligned} \quad (7)$$

Equation (7a-b), show the simple derivation of the augmentation gain used in developing the reported controller. Implementation subtleties which were used include the utilization of the mean magnitude of the augmentation gain and not simply the augmentation gain as computed in (8a-8c).

$$\begin{aligned} ref_{err} &= input_{ref} - output_{nlin} \\ G_{ref} &= output_{nlin} + ref_{err} \\ G_{aug} &= |G_{lin} + G_{ref}|/2 \end{aligned} \quad (8)$$

of course depending on which side of the reference the nonlinear output is situated.

A. *Controller Inference Design Technique For (2)*

Utilizing this knowledge in the direct control of the nonlinear system with all PID controller gains & simulation time staying constant gives the following results for the case when the parameter ***a*** is set to 5. The legend clearly shows the output (considering variable x1) in the linear case and after augmentation of the nonlinear model from knowledge of state error computed earlier in section IV.

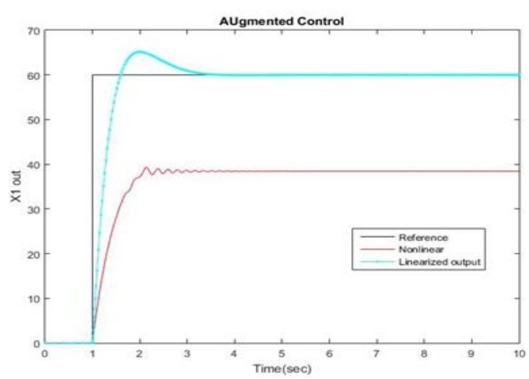

Figure 3. Closed Loop with X1 as feedback & a=5

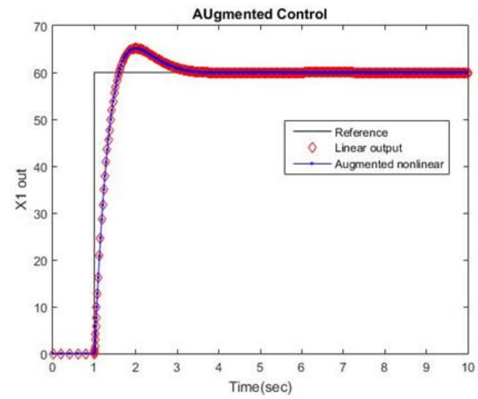

Figure 4. Augmented Output with Linear error @ a=5

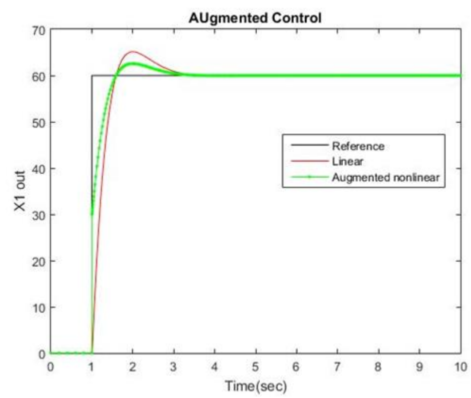

Figure 5. Augmented Output with Reference and Linear error @ a=5

The experiment is repeated when the parameter ***a*** is set to 10;

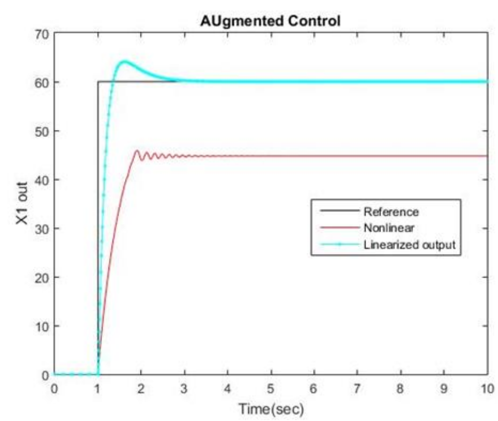

Figure 6. Closed Loop with X1 as feedback & a=10

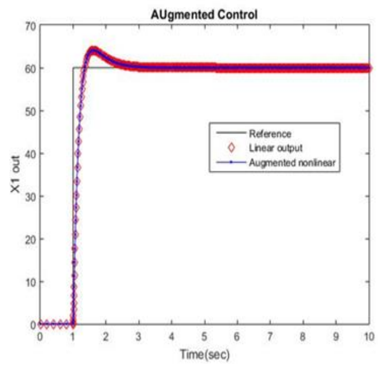

Figure 7. Closed Loop with X1 as feedback & a=10

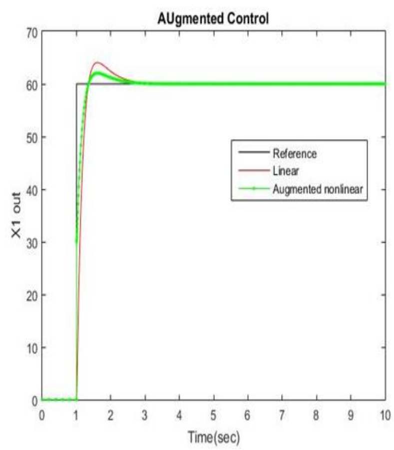

Figure 8. Augmented Closed Loop Response

For the closed loop response Fig. 6-8, when x1 is being controlled, the system settles down quickly with increasing time. The transient oscillation is minimal within the 10sec time considered and the output is sufficiently tracked with minimal steady state error and reduced overshoot.

B. Results for the Cart Driven Inverted Pendulum

The CIP model was implemented and experimented with in simulation. The bounded uncertain parameter was set at 0.5. The following results were obtained when the cart translational position is used as reference input.

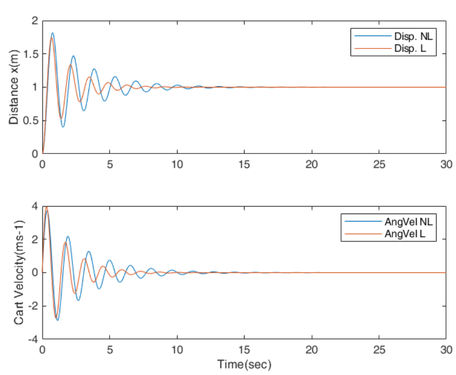

Figure 9. Combined Linear & Nonlinear CIP Response

The plots of the displacement for nonlinear and linear systems are given. This results employed a PID controller tuned by trialing. PID parameters were chosen as 10, 1.0 and 0.1, to give the response in Figure 9. The analysis of the control signal in Figure 10, showed higher magnitude for the control effort in the nonlinear system.

When the augmentation algorithm is implemented on the nonlinear system, the following result was obtained for the translational displacement of the cart, linear velocity, angular displacement of pendulum and angular velocity. Figure 11 shows the nonlinear system after augmentation being a replica of the linear system response as both responses are indistinguishable.

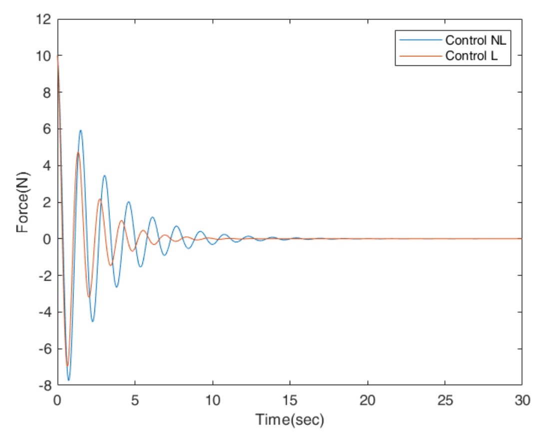

Figure 10. Control effort

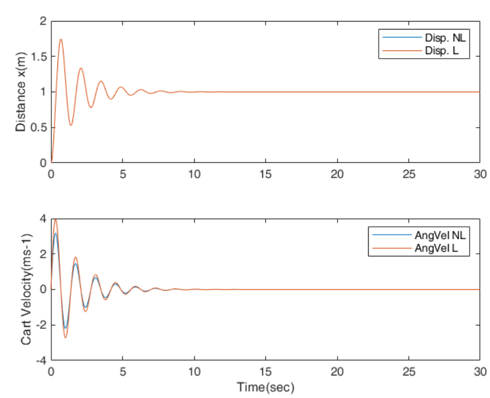

Figure 11. Compensated Augmentation Response

When the reference input as shown in Figure 12 was shifted from a unit step to 5 units step input, Figure 13 shows the response of the nonlinear system still tracking the linear system response

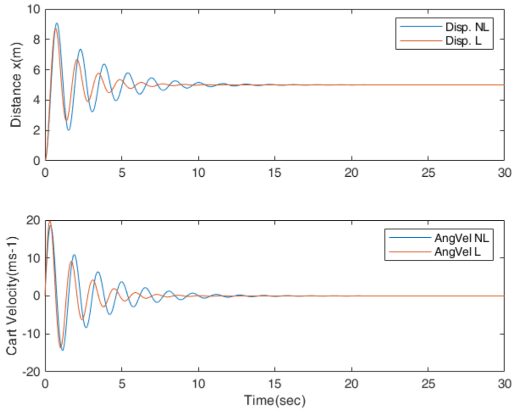

Figure 12. Unit Step Response before Compensation

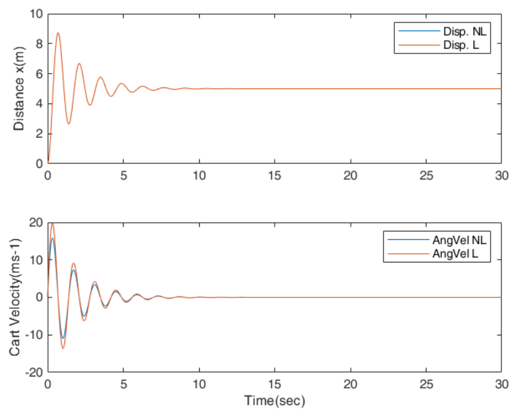

Figure 13. Compensated Augmentation Response

Testing the technique with a reference signal other than step input, a sinusoidal input was injected into the system with the following responses. Figure 14 is the response for sinusoidal reference signal of 1 unit amplitude. Before compensation, the nonlinear response clearly overshot the reference, forming a noticeable transient error.

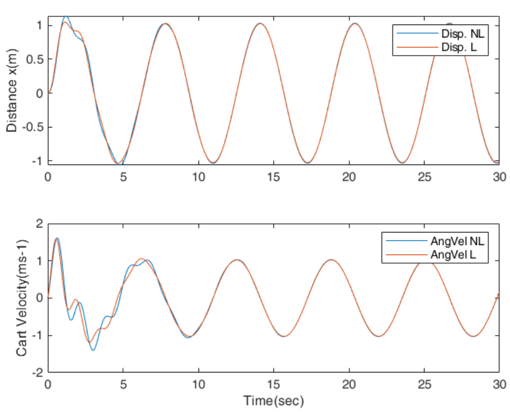

Figure 14. Initial response for Sinusoidal Reference Signal

The control effort for the uncompensated response is as given in Figure 15.

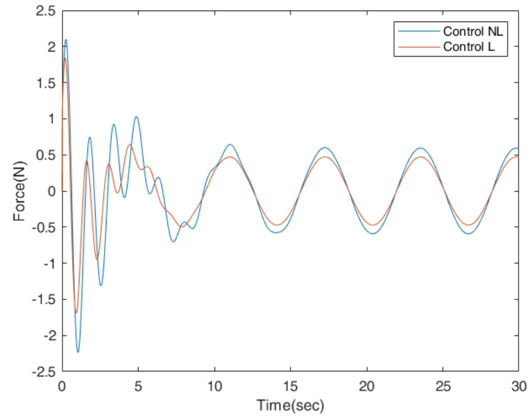

Figure 15. Pre-augmentation Control Effort for Sinusoidal Response

The augmented nonlinear system response is shown in Figure 16. Visible in this plot is the absence of any separation between the linear and nonlinear plots. Such a behavior similar to observer action described in section 1.

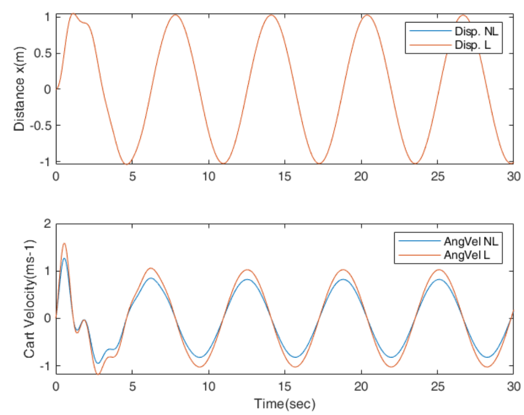

Figure 16. Compensated Response for the Cart Displacement

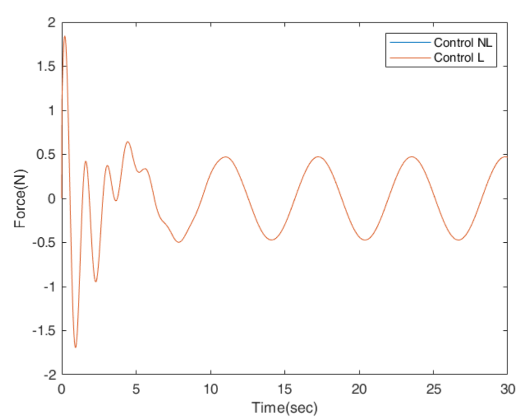

Figure 17. Compensated control effort for Sinusoidal Reference

Seen is the result of the augmentation using the simple algorithm proposed. The control signal (Figure 17) also gives reduced control effort magnitude similar to that obtained for the linear system response.

V. Application of Proposed Output Augmentation Scheme

The proposed method can be applied to the control of both benchmark & generic nonlinear dynamic system models. In hierarchical control architectures, the augmented output regulated control can be implemented for the outer controlled loop of a double loop controlled system. The actuator in the inner loop is assumed to approximately linear, barring the inclusion or consideration of detailed inherent nonlinearities, while the system being controlled has sufficient nonlinear dynamics which can be exploited by this method. Some good examples of physical systems with hierarchical control system models are: Inverted pendulum, Ball and plate System with nonlinear DC motor characteristics, Quad rotor with nonlinear DC motor characteristics and Attitude control system (ACS) of a satellite. This work has tested the proposed method on two nonlinear systems with results being evidence of the workability of the method.

In summary, the same gains utilized in the linear system for the PID are retained in the nonlinear system, with the only modification on the controller coming from the Augmentation gain (7) and (8). The augmentation being made at the output of the otherwise nonlinear system response by enforcing a correction of the nominal nonlinear output with the difference of the linear response and the nominal nonlinear response. The method also showed reduced actuation energy in the nonlinear control signal magnitude leading to more energy efficient control synthesis.

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Appendix

TABLE II. Table of values for CIP [16]

1	Pendulum bob mass(m)	85 or 210g
2	Cart mass(M)	0.49Kg
3	Eccentric length(l)	0.30m
4	Inertia of pendulum	1/3ml^2
4	Damping coefficient(b)	4 to 10 Kgs-1
5	Acceleration due gravity(g)	9.81ms-2
6	Input-to-force gain(Kif)	5N