Kaunia, Vol.XII, Num.1, April 2016, pp. 17~22
ISSN 1829-5266 (paper), ISSN 2301-8550 (online)
Available online at http://ejournal.uin-suka.ac.id/saintek/kaunia

Application of Cutting Stock Problem in Minimizing The Waste of Al-Quran Cover

Noor Saif Muhammad Mussafi

Mathematics Department, Faculty of Science and Technology, UIN Sunan Kalijaga,
Jl. Marsda Adicucipto Yogyakarta 55281, Indonesia
Correspondance; Email: om_norsa@yahoo.com

Abstract

Cutting stock problem is the problem of cutting standard-sized pieces of stock material, such as vinyl rolls (synthetic leather) into pieces of specified sizes while minimizing material wasted. The study of cutting stock problem is expected to provide an alternative solution to publishing industrial. This study examines the cutting stock problem based on illustrative example of Quran Publisher using the integer programming. The problem is transformed into optimization model so that it can be solved using solver menu in Microsoft Excel. The results provide decision that meets the consumer demand and has a minimum waste of raw material that used for Al-Quran cover.

Keywords: Optimization Model, Integer Programming, Cutting Stock Problem.

Introduction

Minimizing wastage is a key factor in improving productivity of a manufacturing plant (Rodrigo et al: 2012). Wastage can occur in many ways and cutting stock problem can be described under the raw material wastage. Therefore, Operations Research plays a major role in minimizing raw material wastage or to maximizing space utilization of the raw material. Many people including scientists have conducted research to overcome above challenges. A cutting stock problem basically describes in two ways, One-Dimensional (1D) and Two-Dimensional (2D) cutting stock problems.

Furthermore, cutting stock problems were first described in the early days of linear programming. In Eiselt and Sandblom (2007:75), Gilmore and Gomory (1961) were the first to formulate cutting stock problems (or, equivalently, stock cutting or trim loss problems). The problem in its simplest form can be described as follows. Given materials that are available in certain shapes and sizes, cut them in order to generate certain desired shapes and sizes, so as to minimize some objective such as cost and waste (Figure 1). Depending on the type of material and cutting under consideration. For instance, cutting newsprint to different lengths is a one-dimensional cutting problem, as the width of the paper remains the same, while only its length varies. In other words, an optimum cutting stock problem can be defined as cutting a main sheet into smaller pieces while minimizing total wastage of the raw material or maximizing overall profit obtained by cutting smaller pieces from the main sheet. Many researchers have worked on the cutting stock problem and developed different algorithms to solve the problem.

Gilmore et al (1961) conducted some of the earliest research in this area and one-dimensional cutting stock problem is solved using Linear Programming Technique. In this study, unlimited numbers of raw materials with different lengths are assumed available in stock, and a mathematical model has been developed to minimize the total cutting cost of the stock length of the feasible cutting patterns and Column Generation Algorithm has been

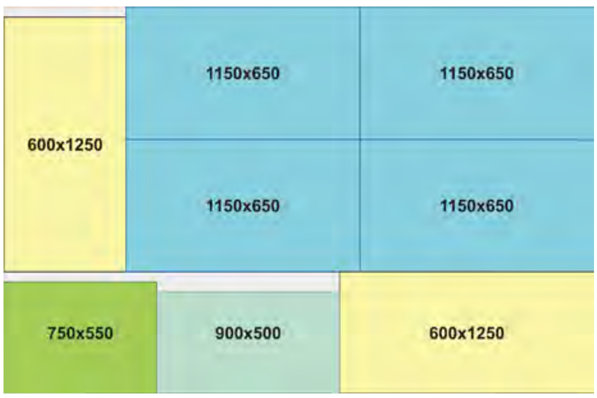

Figure 1 Minimize the waste (unshaded area) in cutting stock problem.

developed to generate feasible cutting patterns. Also, Saad (2001) has modified Branch and Bound Algorithm to find feasible cutting patterns for one-dimensional cutting stock problem and mathematical model has been developed to minimize the total cut loss. In the case study, Saad has selected four different types of steel coils to cut from the standard steel coil with the 130 cm length and width of the main coil and widths of the required coils are equal. Branch and Bound Algorithm has been explained using the example.

Linear Programming and Integer Programming

Since the time it was first proposed by George B. Dantzig in 1947 as a way for planners to set general objectives and arrive at a detailed schedule to meet these goals, linear programming has come into wide use. It has many nonlinear and integer extensions collectively known as the mathematical programming field, such as integer programming, nonlinear programming, stochastic programming, combinatorial optimization, and network flow maximization. In special case, linear programming (LP) plays a role analogous to that of partial derivatives to a function in calculus, it is the first-order approximation. Formally, linear programming is concerned with the maximization or minimization of a linear objective function in many variables subject to linear equality and inequality constraints (Dantzig: 1997).

George Dantzig developed an efficient method, the simplex algorithm, for solving linear programming problems. Since the development of the simplex algorithm, LP has been used to solve optimization problems in industries as diverse as banking, education, industrial, forestry, petroleum, and trucking. In a survey of Fortune 500 firms, 85% of the respondents said they had used linear programming (Winston, 2004:56). Before formally defining a linear programming problem, let's define the concepts of linear function and linear inequality.

Definition 2.1
A function $f(x_1, x_2, \dots, x_n)$ of $x_1, x_2, \dots, x_n$ is a linear function if and only if for some set of constants $c_1, c_2, \dots, c_n$, $f(x_1, x_2, \dots, x_n) = c_1x_1 + c_2x_2 + \dots + c_nx_n$.

Definition 2.2
For any linear function $f(x_1, x_2, \dots, x_n)$ and any number *b*, the inequalities $f(x_1, x_2, \dots, x_n) \leq b$ and $f(x_1, x_2, \dots, x_n) \geq b$ are linear inequalities.

Definition 2.3
A linear programming problem (LP) is an optimization problem that deal with:

a. Maximizing or minimizing a linear function of decision variables. The function that is to be maximized or minimized is called *objective function*.
b. The values of the decision variables must satisfy a set of *constraints*. Each constraints must be a linear equation or linear inequality.
c. A *sign restriction* which associated with each variable. For any variable x_i, the sign restriction specifies that x_i must be either nonnegative ($x_i \geq 0$) or unrestricted in sign (urs).

Definition 2.4
Now consider a system of linear equations: $A \quad = b$
Where $A \in R^{m} \quad, b \in R^{m}, m \geq n$ and $A = n$. Note that the number of unknowns, *n*, is no larger than the number of equations, *m*. Our goal then is to find the vector x minimizing $\|A \quad - b\|^2$.

Theorem 2.5
Let $A \in R^{m} \quad, m \geq n$. Then, $r \quad A = n$ if and only if $r \quad A^TA = n$.
Proof:
The proof provided by Chong and Zak (2001:187).
"$\Rightarrow$": Suppose that $r \quad A = n$. To show $A^TA = n$, it is equivalent to show $N(A^TA) = \{0\}$. To proceed, let $x \in N(A^TA)$; that is, $A^TA \quad = 0$. Therefore,

$$\|A \quad \|^2 = x^TA^TA \quad = 0,$$

which implies that $Ax = 0$. Because $r \quad A = n$, we have $x = 0$.
"$\Leftarrow$": Suppose that $r \quad A^TA = n$; that is $N(A^TA) = \{0\}$. To show $r \quad A = n$, it is equivalent to show that $N(A) = \{0\}$. To proceed, let $x \in N(A)$; that is $A \quad = 0$. Then, $A^TA \quad = 0$, and hence $x = 0$ □

Hurlbert (2010:11) described the following matrix form linear programming which is called as optimization model:

Minimize	c^Tx	(objective function)
Subject to	$A \quad \geq b$	(constraints)
	$x \geq 0$	(sign restriction)

where A is an mxn matrix composed of real entries, $m < n$, $rank\ A = m$, c is constant, and without loss of generality the vectors x and b both are nonnegatives. If component of b is negative, say the i^{th} component, multiply the i^{th} constraint by -1 to obtain a positive right-hand side (Bertsimas and Tsitsiklis: 1997).

Formally Kolman and Beck (1995:51) and Taha (2007) also specified the canonical form of linear programming as follows.

Minimize $\quad z = c_1x_1 + c_2x_2 + \cdots + c_nx_n$
(objective function)

Subject to $c_{1}\ x_1 + c_{1}\ x_2 + \cdots + c_{1n}x_n \ge b_1$
$c_{2}\ x_1 + c_{2}\ x_2 + \cdots + c_{2n}x_n \ge b_2$
....
$c_{m1}x_1 + c_{m2}x_2 + \cdots + c_{m}\ x_n \ge b_m$
(constraints)

$x_j \ge 0, j = 1,2, \ldots, n$
(sign restriction)

On the other hand, one key limitation that prevents many more applications is the assumption of divisibility, which requires that noninteger values be permissible for decision variables. In many practical problems, the decision variables actually make sense only if they have integer values. For example, it is often necessary to assign people, machines, and vehicles to activities in integer quantities. The mathematical model for integer programming is the linear programming model with the one additional restriction that the variables must have integer values (Hillier and Lieberman, 2001:576).

Excel Solver

Microsoft Excel has the capability to solve linear (and often nonlinear) programming problems, which is called excel solver. The key to solving an LP on a spreadsheet is to set up a spreadsheet that tracks everything of interest (costs or profits, resource usage, etc.). Next, identify the cells of interest that can be varied. These are called changing cells. After defining the changing cells, identify the cell that contains objective function as the target cell. Next, identify the constraints and tell the Solver to solve the problem. At this point, the optimal solution to our problem will be placed in the spreadsheet. Winston (2004:203) introduces the steps of Solver to find the optimal solution:

a. After the optimization model defined (decision variables, objective function, constraints, and sign restriction), arrange the matrices of such problem into spreadsheet (Figure 2).
b. From the Tools menu, select Solver. The dialog box in Figure 3 will occur.
c. Move the mouse to the Set target Cell portion of the dialog box and click (or type in the cell address) on target cell (total cost) and select Min. This tells Solver to minimize total cost.
d. Move the mouse to the By Changing Cells portion of the dialog box and click on the changing cells. This tells Solver it can change the amount subject.

	A	B	C	D	E	F	G	H
1	INTEGER PROGRAMMING OF PROBLEM "X"							
2								
3								
4	DECISION VARIABLES							
5		Variable 1	Variable 2	....	Variable n			
6	Solution							
7								
8	OBJECTIVE FUNCTION							
9		Variable 1	Variable 2	....	Variable n			
10	Coefficients							
11								
12	Total							
13								
14	CONSTRAINTS							
15		Variable 1	Variable 2	....	Variable n	Totals	Sign	Required
16	Constraint 1							
17	Constraint 2							
18	....							

Figure 2 Problem arrangement.

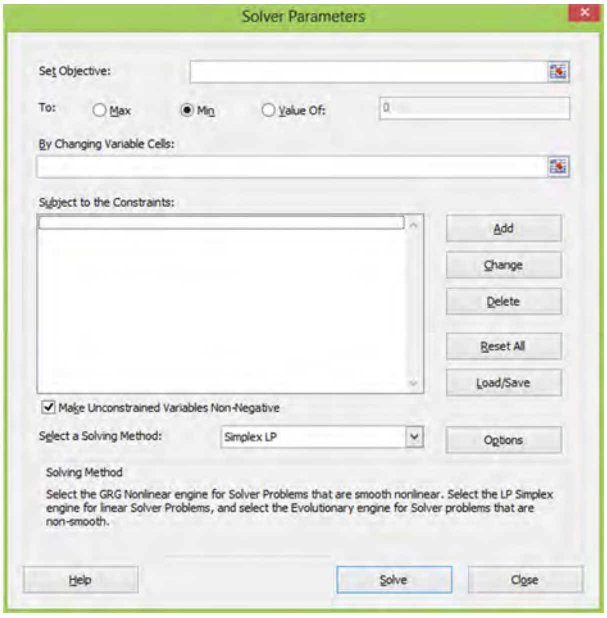

Figure 3 Dialog box of Solver.

Click on the Add button to add constraints. The screen in Figure 4 will appear. Move to the Cell reference part of the Add Constraint dialog box and select the cells. Then move to the dropdown box and select >=. Add any constraints depend on the problem. Choose OK if there are no more constraints. In case of integer programming, add new constraint that provide the solution equals to integer.

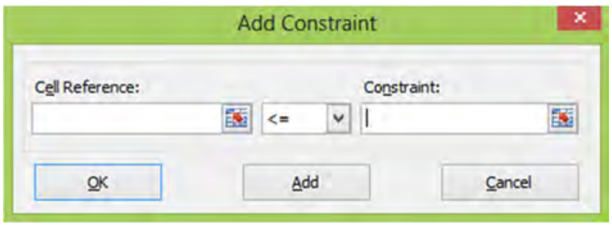

Figure 4 Dialog box for adding constraints.

e. Before solving the problem, we need to tell Solver that all changing cells must be nonnegative. We must also tell Solver that we have a linear model. Choose solving method of simplex LP then click OK (Figure 4). After choosing OK from the Solver Options box, we then select Solve and it will yields the optimal solution (Figure 3).

Methods and Illustrative Example

Prior to finding raw material wastage of one-dimensional cutting stock problem, it needs the following steps:

(Step 1) Identify the elements of such problem: the objective, variables, constraints, and sign restriction.
(Step 2) Construct the optimization model using the concept of integer programming, that is minimizing the objective function subject to constraints.
(Step 3) Set up the matrix and vector of such model into excel spreadsheet based on integer programming spreadsheet.
(Step 4) Using the toolbox excel solver consider the cells correspond to the Set Objective, the Goal (minimize), By Changing Variables, and the Constraints.
(Step 5) Select Solving Method of Simplex LP and then execute Solve to acquire optimal solution.

An illustrative example

A private company that concern in the field of Al-Quran publishing namely Al-Jumu'ah sells four types of Al-Quran with various sizes in centimeters:

a. Al-Quran Alfa (21 x 29)
b. Al-Quran Beta (14 x 20)
c. Al-Quran Gamma (10 x 14)
d. Al-Quran Kappa (7 x 10)

One day Al-Jumu'ah received consumer order of Al-Quran Alfa, Al-Quran Beta, Al-Quran Gamma, and Al-Quran Kappa consecutively 20 units, 15 units, 8 units, and 10 units. In order to meet consumer demand, i.e. needs of cover of Al-Quran, the company should buy vinyl fabric materials that are known to have a variety of sizes, the large vinyl fabric (2,058 cm2), the medium vinyl fabric (1,625 cm2), and the small vinyl fabric (900 cm2). How to determine the proper purchase of vinyl fabric to minimize the remaining pieces so that satisfy consumer demand correspond to Table 1?

Table 1 Size of Al-Quran and its demand.

Size of Al-Quran	Number of Vinyl Fabric			Demand
	Large (2035 cm^2)	Medium (1.250 cm^2)	Small (700 cm^2)	
21X29 cm	2	0	1	20
14X20 cm	1	3	1	15
10X14 cm	1	4	0	8
7X10 cm	6	2	0	10

Results

The cutting stock problem has been discussed in various application, mostly in terms of industrial manufacture. This research will try to solve the above illustrative example.

***(Step 1)** Identify the elements of such problem: the objective, variables, constraints, and sign restriction.*

Al-Jumu'ah must decide the combination of purchasing vinyl fabric to yield a minimum waste. So the objective clearly to minimize the number of vinyl fabric that must be spent. Suppose $x_1, x_2,$ and x_3 consecutively correspond to large vinyl, medium vinyl, and small vinyl. Hence the objective function of Al-Jumu'ah is

$$\text{Minimize } z = x_1 + x_2 + x_3.$$

This means that Al-Jumu'ah can minimize its total waste by minimizing the number of vinyl fabric that are cut.

Al-Jumu'ah faces the following four constrains: Constraint 1 At least 20 units of Al-Quran Alfa must be made. Constraint 2 At least 15 units of Al-Quran Beta must be made. Constraint 3 At least 8 units of Al-Quran Gamma must be made. Constraint 4 At least 10 units of Al-Quran Kappa must be made. Because the total number of Al-Quran Alfa that are made is given by $2x_1 + x_3$

constraint 1 becomes $2x_1 + x_3 \geq 20$
Similarly, constraint 2 becomes $x_1 + 3x_2 + x_3 \geq 15$
also constraint 3 becomes $x_1 + 4x_2 \geq 8$
and constraint 4 becomes $6x_1 + 2x_2 \geq 10$

Note that the x_i for $i = 1,2,3,\ldots,6$ should be required to assume nonnegative integer values, that is $x_i \geq 0$.

(Step 2) *Construct the optimization model using the concept of integer programming, that is minimizing the objective function subject to constraints.*

By combining the above information, we could construct the optimization model of integer programming as follow:

Minimize $z = x_1 + x_2 + x_3$

Subject to
$$2x_1 + x_3 \geq 20$$
$$x_1 + 3x_2 + x_3 \geq 15$$
$$x_1 + 4x_2 \geq 8$$
$$6x_1 + 2x_2 \geq 10$$
$$x_1, x_2, x_3 \geq 0$$

(Step 3) *Set up the matrix and vector of such model into excel spreadsheet based on linear programming spreadsheet.*

Based on the above optimization model, we can consider matrix *A* and vector *b*

$$A = \begin{bmatrix} 2 & 0 & 1 \\ 1 & 3 & 1 \\ 1 & 4 & 0 \\ 6 & 2 & 0 \end{bmatrix} \text{ and } b = \begin{bmatrix} 20 \\ 15 \\ 8 \\ 10 \end{bmatrix}$$

The integer programming spreadsheet as shown in Figure 2 could be applied for above matrix (cells B15 to D18) and vector (cells G15 to G18) so that it will appear Figure 5.

	A	B	C	D	E	F	G
1	INTEGER PROGRAMMING OF PROBLEM "WASTE OF AL-QURAN COVER"						
2							
3	DECISION VARIABLES						
4		x_1	x_2	x_3			
5	Solution						
6							
7	OBJECTIVE FUNCTION						
8		x_1	x_2	x_3			
9	Coefficients	1	1	1			
10							
11	Total (Z)						
12							
13	CONSTRAINTS						
14		x_1	x_2	x_3	Totals	Sign	Required
15	Constraint 1	2	0	1		>=	20
16	Constraint 2	1	3	1		>=	15
17	Constraint 3	1	4	0		>=	8
18	Constraint 4	6	2	0		>=	10

Figure 5 Al-Jumu'ah IP Problem.

(Step 4) *Using the toolbox excel solver consider the cells correspond to the Set Objective, the Goal (minimize), By Changing Variables, and the Constraints.*

Set cell B11 as Objective and choose the goal of minimize. Moreover By Changing Variables is fullfiled by cells between B5 to D5 as a set of solution. Also, define the Four Constraints (E15>=G15, E16>=G16, E17>=G17, and E18>=G18) and one additional Constraint (B5 to D5 equal to integer) to yields nonnegative integer solution as shown in Figure 6.

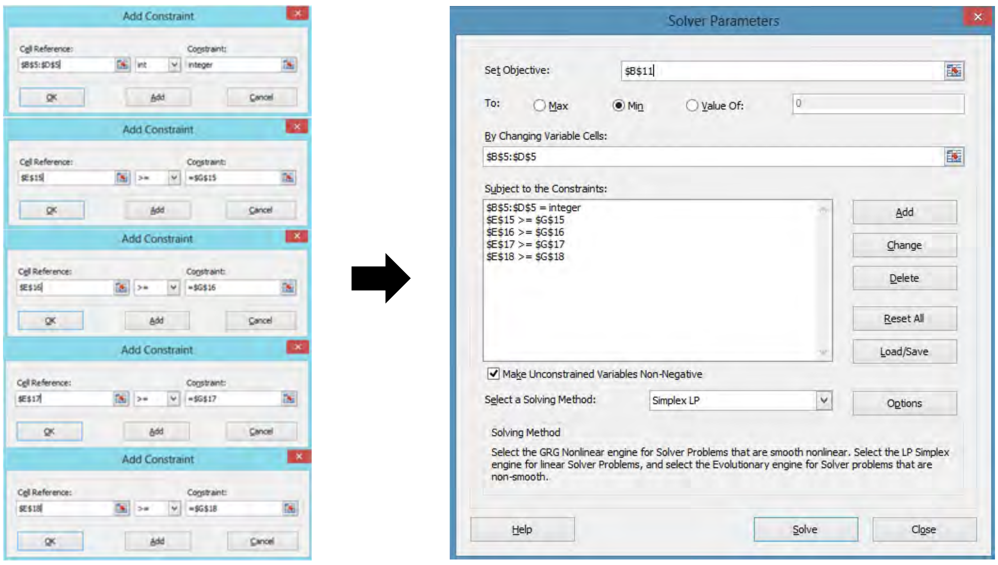

Figure 6 Solver Parameter of Al-Jumu'ah.

(Step 5) *Select Solving Method of Simplex LP and then execute Solve to acquire optimal solution.*

Make sure that we select a Solving Method of Simplex LP (Figure 6) and click solve to attain optimal solution as displayed in Figure 7.

	A	B	C	D	E	F	G
1	**INTEGER PROGRAMMING OF PROBLEM "WASTE OF AL-QURAN COVER"**						
2							
3	**DECISION VARIABLES**						
4		x_1	x_2	x_3			
5	**Solution**	10	2	0			
6							
7	**OBJECTIVE FUNCTION**						
8		x_1	x_2	x_3			
9	Coefficients	1	1	1			
10							
11	**Total (Z)**	12					
12							
13	**CONSTRAINTS**						
14		x_1	x_2	x_3	Totals	Sign	Required
15	Constraint 1	2	0	1	20	>=	20
16	Constraint 2	1	3	1	16	>=	15
17	Constraint 3	1	4	0	18	>=	8
18	Constraint 4	6	2	0	64	>=	10

Figure 7 Optimal Solution.

Figure 7 suggests Al-Jumu'ah to purchase 10 units large vinyl fabric (cell B5) and 2 units medium vinyl fabric (C5) to meet the customer demand of 20 units Al-Quran Alfa, 15 units Al-Quran Beta, 8 units Al-Quran Gamma, and 10 units Al-Quran Kappa.

Conclusion

In this study, a cutting stock problem is formulated as optimization model based on integer programming concept. Such problem also can be solved properly using Excel Solver. As given in Figure 7, this methods propose Al-Jumuah to purchase $Z_m = 12$ units vinyl fabric, by considering x_1 = large = 10 unit and x_2 = medium = 2 unit, in order to satisfy the costumer demand and to minimize the piece of waste of Al-Quran cover. In this case study, the plant assumes that all the extra pieces from each item as waste so that it can be reduced as much as possible. The minimum waste yield two benefits at once, that increase Al Jumuah's profits and support eco-friendly programs.

References

D. Bertsimas and J.N. Tsitsiklis. (1997). Introduction to Linear Optimization. 1st edition. Athena Scientific Belmont Massachusetts.

George Dantzig. (1997). Linear Programming 1: Introduction. 2nd edition, Springer Verlag–New York.

Glenn H. Hurlbert. (2010). Linear Optimization: The Simplex Workbook. Springer Science + Business Media. New York.

H. A. Eiselt and C.-L. Sandblom. (2007). Linear Programming and its Applications. 1st edition. Springer Verlag.

Hamdy A. Taha. (2007). Operations Research: An Introduction. 8th edition. Prentice Hall. Upper Saddle River N.J.

Hillier and Lieberman. (2001). Introduction to Operations Research. 7th edition, McGraw Hill Companies.

P. C. Gilmore and R. E. Gomory. (1961). A Linear Programming Approach To The Cutting Stock Problem. Operations Research. Vol. 9 No. 2: 849 -859.

P. C. Gilmore and R. E. Gomory. (1963). A Linear Programming Approach To The Cutting Stock Problem –PartII. Operations Research. Vol. 11 No. 6: 863 – 888.

Saad M. A. Suliman. (2001). Pattern generating procedure for the cutting stock problem. International Journal of Production Economics. Vol. 74: 293-301.

W. N. P Rodrigo, W.B. Daundasekera, and A.A.I Perera. (2012). Pattern Generation for Two-Dimensional Cutting Stock Problem. International Journal of Mathematics Trends and Technology (IJMTT). Vol. 3 Issue 2: 54-62.

Wayne L. Winston. (2004). Operations Research, Applications and Algorithms. 4th edition. Duxbury Press Thomson Learning, Inc.