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The Effect of Initial Intensity of Light Sources on the Fraunhofer Diffraction Pattern for Circular Apperture Using the Gauss-Legendre 4 Point Quadrature Integration Numerical Method: Scientific and Islamic Integration-Interconnection

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Abstract

Visualization of Fraunhofer diffraction pattern of circular apperture has been obtained using the 4-point Gauss-Legendre Quadrature integration method. The 4-point Gauss-Legendre quadrature integration method is used to solve the integral function of intensity. The intensity function is used to display the intensity distribution of the Fraunhofer diffraction pattern of circular apperture in a visual form through a computer programming simulation based on GUI (Graphycal User Interface) MATLAB. Simulation results show changes in initial intensity affect diffraction patterns.

***Keywords**: Fraunhofer diffraction, Gauss-Legendre 4 point.*

Introduction

Many researches has been carried out on various things from the beginning of the development of science to the present. The virtue of doing research and understanding of everything that happens is mentioned in the Qur'an, found in Surah Al-Imran verse 190 which reads:

إِنَّ فِي خَلْقِ ٱلسَّمَٰوَٰتِ وَٱلْأَرْضِ وَٱخْتِلَٰفِ ٱلَّيْلِ وَٱلنَّهَارِ لَءَايَٰتٍ
لِّأُو۟لِى ٱلْأَلْبَٰبِ ١٩٠

Meaning: "Verily in the creation of heaven and earth, and the alternation of night and day there are signs (the greatness of God) for intelligent people" (Tim Penerjemah, 2004).

Research arises from curiosity and ideas on various phenomena that occur both from simple to complex. Various categorized research objects are very diverse, especially in physics, both in very large sizes such as planets, stars, until the size exceeds the solar system, to a very small size such as particles, atoms, to light that is useful to help vision. Vision is something that is very valuable for living things. Natural phenomena that occur in this world can be observed through visions such as color, brightness, darkness, shape, and many more. Vision can occur due to optical phenomena. Optical phenomenon means that it is a phenomenon that involves the behavior and nature of light and how it interacts with matter (Hecht, 2002). Vision can occur due to the visible light spectrum reflected to the eye. Without light, vision will not occur. Allah says in Q.S Al-Baqarah Verse 20:

يَكَادُ ٱلْبَرْقُ يَخْطَفُ أَبْصَٰرَهُمْ ۖ كُلَّمَآ أَضَآءَ لَهُم مَّشَوْا۟ فِيهِ وَإِذَآ أَظْلَمَ
عَلَيْهِمْ قَامُوا۟ ۚ وَلَوْ شَآءَ ٱللَّهُ لَذَهَبَ بِسَمْعِهِمْ وَأَبْصَٰرِهِمْ ۚ إِنَّ ٱللَّهَ عَلَىٰ
كُلِّ شَىْءٍ قَدِيرٌ ٢٠

Meaning: "Almost the flash grabbed their vision. Every time the flash illuminated them, they walked under the light, and when darkness came upon them, they stopped. If God wills, He will obliterate their hearing and vision. Truly, God has power over all things" (Tim Penerjemah, 2004)

The verse explains that God can reduce the light so that humans can see and can take back the light so that humans cannot see. Allah has power over the vision of his creatures. To give thanks for the pleasure of vision, it is better for humans to use vision for things that are useful to humanity, one of which is to examine optical phenomena to add to the knowledge that is useful to humans.

There is an optical phenomenon when a ray passes through an obstacle, it will deflect and produce a dark and bright pattern. Around the 1600's, Francesco Grimaldi studied the phenomenon and named it diffraction (Hecht, 2002). There are two types of diffraction, namely Fresnel and Fraunhofer diffraction. There are many types of diffraction patterns based on the shape of the gap, however the diffraction pattern with a circular apperture has a significant effect in the development of optical instruments (Tipler, 2004).

The diffraction pattern can be observed based on its intensity distribution. The intensity of the Fraunhofer diffraction pattern can be expressed in mathematical form (Widagda, 2015). The Fraunhofer diffraction pattern intensity distribution of the circular apperture can be visualized based on its mathematical model, but it takes a lot of calculations because of the many points that the intensity must be calculated.

One way to calculate the number of calculations is to use a computer. With the help of a computer, calculations can be completed more easily and quickly. But to solve complicated mathematical forms computers need numerical methods (Munir, 2010).

There are three numerical methods approaches that are used to solve integral functions, namely the pias method, Newton-Cotes, and quadrature-Gauss (Munir, 2010). In the Gauss quadrature method, the calculation of the integral value is done by taking the value of the function at a certain point (fixed point) called the evaluation point and multiplying with the integration weighting function (Darmawan, 2016).

Gauss quadrature method has a higher accuracy than other methods (Prasetya, 2016). There are many types of Gauss Quadrature, but the Gauss-Legendre Quadratur has the highest accuracy (Darmawan, 2016). Therefore, the Gauss-Legendre quadrature method is considered better in solving mathematical models of the Fraunhofer diffraction pattern of circular apperture.

Simulation of Fraunhofer diffraction for circular apperture using the 4 points Gauss-Legendre quadrature integration method needs to be done because there has never been any research on Fraunhofer diffraction pattern of circular apperture using this method.

Light Intensity of Fraunhofer diffraction for circular apperture

Circle apperture diffraction is one of diffraction whose gap is in the form of area, so that the intensity distribution of diffraction pattern requires double integration. The function can be calculated using an expansion series, and the easiest way to express the result is to quote the actual number obtained from the expansion (Jenkins, 2000), that is:

$$\tilde{E} = \frac{\varepsilon_A}{R} e^{i(\omega t - kR)} \int_{\rho=0}^{a} \int_{\phi=0}^{2\pi} e^{i\left(\frac{k\rho q}{R}\right)\cos(\phi-\Phi)} \rho \, d\rho \, d\phi, \quad (1)$$

with $\Phi = 0$, then the integral equation is a ϕ function become,

$$\int_{\phi=0}^{2\pi} e^{i\left(\frac{k\rho q}{R}\right)\cos(\phi-\Phi)} \, d\phi. \quad (2)$$

Equation (2) can no longer be simplified to more general forms such as trigonometric, exponential or hyperbole functions (Hecht, 2002). However, the equation is identical to the m order Bessel function that is shaped:

$$J_m(u) = \frac{i^{-m}}{2\pi} \int_0^{2\pi} e^{i(mv + \cos v)} \, dv, \quad (3)$$

So that equation (1) can be formulated,

$$\tilde{E} = \frac{\varepsilon_A}{R} e^{i(\omega t - kR)} 2\pi \int_{\rho=0}^{a} J_0\left(\frac{k\rho q}{R}\right) \rho \, d\rho. \quad (4)$$

J_0 can be changed to J_1 using the recursion relationship so it produces,

$$\int_0^u u' J_0(u') \, du' = u J_1(u), \quad (5)$$

with variables changed by assume $w = \frac{k\rho q}{R}$, then equation (4) becomes,

$$\tilde{E} = \frac{\varepsilon_A}{R} e^{i(\omega t - kR)} 2\pi a^2 \left(\frac{R}{kaq}\right) J_1\left(\frac{kaq}{R}\right). \quad (6)$$

The intensity at P point is $\langle (\mathrm{Re}\, E)^2 \rangle$ or $\frac{1}{2} E E^*$. The intensity of the light at P point is:

$$I = \frac{2{\varepsilon_A}^2 A^2}{R^2} \left[\frac{J_1\left(\frac{kaq}{R}\right)}{\frac{kaq}{R}}\right]^2, \quad (7)$$

where A is the circle area at the center of diffraction pattern. The intensity at the center of diffraction pattern (P_0) can be determined by adjusting q = 0 so that $J_1(0)$ = 0, but $\frac{J_1(u)}{u} = \frac{1}{2}$ for u = 0 (Hecht, 2002). Then, the intensity at P_0 point can be formulated,

$$I_0 = \frac{{\varepsilon_A}^2 A^2}{2R^2}. \quad (8)$$

If in the above equation R is considered constant and $\sin\theta = \frac{q}{R}$, then equation (7) can be written in the θ function to be

$$I(\theta) = I_0 \left[\frac{2J_1(ka\sin\theta)}{ka\sin\theta}\right]^2, \quad (9)$$

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use the relationship in equation (3), then J_1 can be formulated into,

$$J_1\left(\frac{kaq}{R}\right) = \int_0^{\pi} \cos\left(\theta - \frac{kaq}{R}\sin\theta\right) d\theta \qquad (10)$$

The 4 Points Gauss-Legendre Quadrature Integration Method

Gauss Quadrature integration method is a method that does not use a lot of area division, but utilizes the emphasis and integration weighting. The area that is calculated in this method is the area under the straight line between -1 to 1 (Munir, 2010).

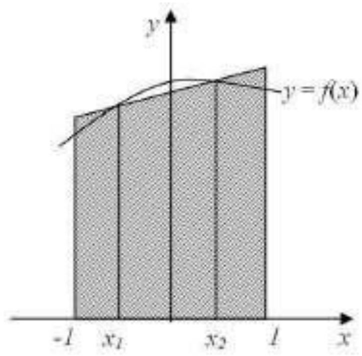

Figure 1. Gauss Quadrature Integration Method (Prastya, 2016).

Figure 1 is an illustration of the concept for the Gauss quadrature integration. Based on the picture, the area of *f(x)* with a limit of −1 to 1 can be approximated by the following equation: (Darmawan, 2016)

$$\int_{-1}^{1} f(x)\,dx = A_1 f(x_1) + A_2 f(x_2), \qquad (11)$$

x_1 and x_2 is an evaluation point at which point the approach line must be tangent to the actual line at the extent of the integral. A_1 and A_2 is the length of the interval specified or called the weighting function (Darmawan, 2016). For points that have amounts *i*, integration can be formulated: (Munir, 2010)

$$\int_{-1}^{1} f(x)\,dx = A_1 f(x_1) + A_2 f(x_2) + A_3 f(x_3) + \cdots + A_i f(x_i), \qquad (12)$$

value *i* is the number of evaluation points and weighting functions. The greater of value *i*, smaller error and more accurate the calculation (Darmawan, 2016).
Quadrature Integration of 4 Points Gauss-Legendre has 4 evaluation points, so the integration can be formulated:

$$\int_{-1}^{1} f(x)\,dx = A_1 f(x_1) + A_2 f(x_2) + A_3 f(x_3) + A_4 f(x_4), \qquad (13)$$

Evaluation point x_i used in the Gauss-Legendre quadrature is obtained from the root of the Legendre polynomial solution and the number of evaluation points needed is obtained by order (*m*) from Legendre polynomial. For *m* = 4 then obtained,

$$P_4(x) = \frac{1}{8}(35x^4 - 30x^2 + 3), \qquad (14)$$

From equation (14) the roots are obtained which will be the evaluation points. Weighting function (A_i) on the integration of the 4 point Gauss Legendre quadrature is determined through equation (13) and can be formulated:

$$A_i = \frac{2}{(1-x_i^2)\left(P_4'(x_i)\right)^2}. \qquad (15)$$

By substituting the results of equations (14) and (15) to equation (13), they are obtained.

$$\int_{-1}^{1} f(x)\,dx = 0{,}3478 f(-0{,}8611) + 0{,}6521 f(-0{,}3400) + 0{,}6521 f(0{,}3400) + 0{,}3478 f(0{,}8611). \qquad (16)$$

Materials and Methods

The object in this research is the Fraunhofer diffraction with the circular apperture as a barrier. The diffraction pattern is formed through the program with the variable vary is the initial intensity (I_0). In making this program used GUI (*Graphical User Interface*) Matlab.

The numerical progression steps are displayed in the flow chart in Figure 2.

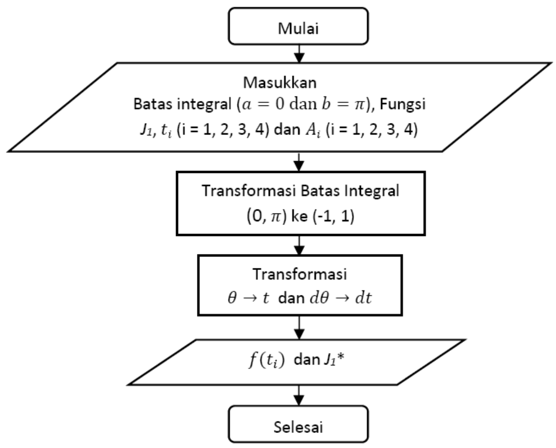

Figure 2. Flow chart of numerical processing.

The stages of visualization of diffraction patterns are displayed in the flow chart in Figure 3.

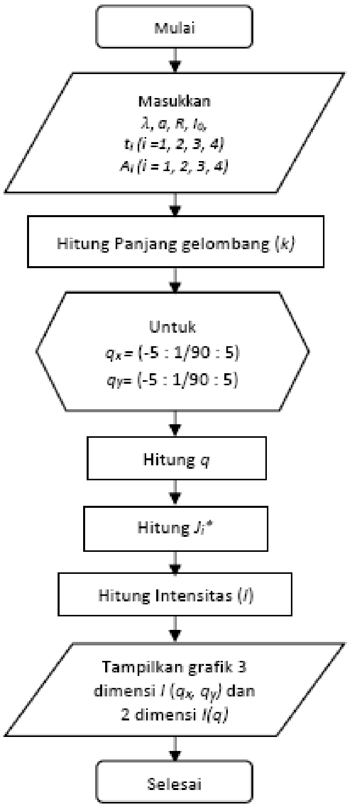

Figure 3. Flow chart of the diffraction patterns visualization processing.

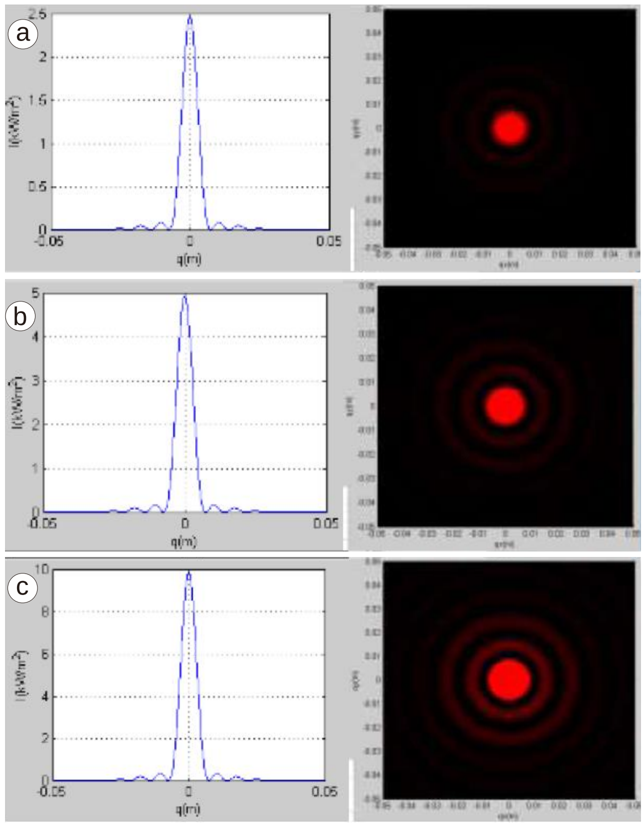

Figure 4. Visualization of the Fraunhofer diffraction pattern for circular apperture with variations in Initial Intensity (I_0) (A) 1 kW/m^2; (B) 2 kW/m^2; dan (C) 4 kW/m^2.

Result and Discussion

The Fraunhofer diffraction visualization results for the circular apperture using the GUI Matlab for the initial intensity change effect with the value a = 0,04 mm, R = 2 m and λ = 700 nm (Yanuarief, 2016).

In equation (9), a mathematical model of the light point intensity as it falls onto the screen (I) influenced by initial intensity ($I0$). So that the visualization of the Fraunhofer diffraction circular apperture intensity distribution is influenced by the initial intensity. Figure 4 shows that the initial intensity difference only affects the peak intensity value. Greater initial intensity results in higher intensity of diffraction patterns seen from the center of the circle and bright rings are getting brighter while smaller initial intensities result in lower intensity diffraction patterns seen from the center of the circle and bright rings that are getting fainter.

This research departs from the knowledge of how the properties of light can be observed by humans. One of the properties of light that can be directly observed by humans is that light can be reflected. The reflection of the light can enable humans to see what is in front of them. The reference to reflected light is already contained in Q.S Al-Baqarah Verse 20 which was mentioned in the introduction above.

According to the Quraish Shihab Interpretation, the lightning bolt had nearly captured their vision and with the help of its light they stepped. As the light fades and the darkness becomes darker they cease to be confused and lost. In accordance with these interpretations, the existence of this research may show that what has been said in the Qur'an which has come down from ancient times is not in conflict with the results of research in modern times. Research related to optical phenomena continues with various approaches to solving problems. The results of this research provide confirmation of the verses of the Qur'an that have been listed. The integration-interconnection model in this case is the confirmative model.

Conclusion

The intensity of the Fraunhofer diffraction pattern for the circular apperture is lower if the initial intensity is low and becomes higher if the initial intensity is high.

References

Brand, J. C. D. 1995. *Lines of Light: The Sources of Dispersive Spectroscopy, 1800–1930*. CRC Press. ISBN 978-2-88449-163-1.

Bruno, T. J. dan Svoronos, P. D. N. 2006. *Handbook of Fundamental Spectroscopic Correlation Charts*. CRC Press. ISBN 0-8493-3250-8.

Dermawan, R. 2016. Perbandingan Metode Gauss Legendre, Gauss Lobatto dan Gauss Kronrod Pada Integrasi Numerik Fungsi Eksponensial. *Jurnal Matematika dan Pendidikan Matematika,* **Vol. I No. 2.**

Firausi, K.S., Sutini, dan Setiabudi, W. 2003. Difraksi Fraunhofer Sebagai Metode Alternatif Sederhana Spektrum. *Berkala Fisika,* **Vol 6. No.1 Hal 1-4.** ISSN 1410-9662.

Halliday, D., dan Resnick, R. 2015. *Fundamental of Physics* (10th ed*).* Wiley, ISBN 978-1-118-23072-5.

Hecth, E. 2002. *Optics* (4th ed*)*. Addison-Wesley Publishing Company. Canada.

Jenkins, F.A., dan White, H.E. 2001, *Fundamentals of Optics* (4th ed). McGraw-Hill International Book Company. Tokyo. ISBN 0-07-256191-2.

Munir, R. 2010. *Metode Numerik.* Informatika Bandung. Bandung.

Prasetia, A. 2016. Performansi Metode Trapesium Dan Metode Gauss-Legendre dalam Penyelesaian Integral Tertentu Berbantuan Matlab. *Jurnal Mercumatika,* **Vol. 1 No. 1** ISSN: 2548-1819.

Starr, C. 2005. *Biology: Concepts and Applications*. Thomson Brooks/Cole. ISBN 978-0-534-46226-0.

Scheid, F. 1968. *Numerical Analysis* (2nd ed). McGraw-Hill. ISBN 07-055197-9.

Tim Penerjemah. 2004. *Al-Qur'an dan Terjemahannya*. Jakarta: Departemen Agama RI.

Tipler, P. A dan Mosca, G. 2004. *Physic for Scientist and Engineers* (5th ed). W.H, Freeman and Company. Newyork. ISBN: 0-7167-4389-2.

Widagda, I. G. A. 2015. Simulasi Intensitas Difraksi Pada Celah Lingkaran (Circuler Aperture) dengan Metode Simpson, *Jurnal Universitas Udayana*.

Yanuarif, C. 2016. Simulasi Pola Difraksi Fraunhofer untuk Celah Lingkaran dengan Modifikasi Fungsi Bessel. *Jurnal Integrated Lab UIN Sunan Kalijaga,* **Vol. 4 No. 2.** ISSN 2339-0905**.**

Zhang, Z., Bai, H., Yang, G., Jiang, F., Ren, Y., Li, J., Yang, K., dan Yang, H. 2013. Computer Simulation of Fraunhofer Diffraction Based on MATLAB. *Optik,* **Vol. 124 Issue 20 Pages 4449-4451.** ISSN 0030-4026.