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Artificial Bee Colony Algorithm to Solve the Shortest Route Problem with Fuzzy Arc Weight

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Abstract

The shortest path problem usually assumes a clear value (crisp) for the weights of each route. However, the crisp weight sometimes ends up ambiguous in practice in the daily life. The number of arc weights is calculated using fuzzy logic, α-cut fuzzy numbers. The Artificial Bee Colony (ABC) algorithm that adopts bee behavior in food searching is used to solve the shortest path problem. This study discusses how to solve numerical problems to find the shortest path using the Artificial Bee Colony algorithm if the arc weights are fuzzy numbers. The algorithm starts with finding the initial solution using Algorithm 1 and then calculated each distance using the sum of α-cut methods. After that, do a local search for each initial solution using the genetic algorithm mutation operator, then searched the distance amount using the same way then compared using the result of distance $D_{2,1/2}$. *The next step was to calculate the fitness value of each solution that would be used to calculate the probability value. The final step was to improve the solution, and an improved solution is said to be a solution if it does not improve again anymore. The calculation process was done repeatedly from the second step to the maximum iteration, i.e. when iteration had reached the limit or the iteration limit fails. Based on the calculation process using ABC algorithm in the case of numerical example, delivery of clean water supply in Gunung Kidul Regency, obtained the shortest route, that is route 1,2,3,5,6 with interval distance equal to 459, 9142.*

***Keywords**: Shortest Path Problem, Fuzzy Trapezoid Number, α-cut Fuzzy Number, Artificial Bee Colony Algorithm, Genetic Algorithm.*

Introduction

The Artificial Bee Colony algorithm was designed by Karaboga in 2005 (Karaboga, 2005) which was inspired by the social behavior of a bee colony where a bee can reach a food source with the closest route. This algorithm has 3 main components in finding the best food source, namely employed bees, onlooker bees and scout bees.

The development of mathematics in the 20th century was relatively fast so that it increasingly met the needs of mathematics users in supporting the development of science and technology. One of the mathematical materials that is developing quite rapidly is logic, sets, and fuzzy numbers. Not all groups encountered in everyday life are clearly defined, for example, a group of smart students, a distance of about 50 km, a car speed of about 70 km/hour, and a rather hot weather. This example is different from him having 3 children, his weight is 50 kg, and the exact distance is 300 m which is an example of a firm or crisp set.

Fuzzy logic was first introduced by Lotfi Zadeh in 1965. The fuzzy shortest path problem has previously been discussed by Klein (1991), Okada and Gen (1994). In this study, the Artificial Bee Colony algorithm is used to find the shortest fuzzy route or path. The Artificial Bee Colony Algorithm is an algorithm that is inspired by the social behavior of a bee colony.

Artificial means that the solution using the Bee Colony algorithm is not exactly the same as the behavior of the bee colony or in other words this algorithm uses several steps that the bee colony does with the help of other algorithms. Based on previous studies, this study would complete the search for the shortest route from a numerical problem with weights from one point to another in the form of fuzzy numbers or fuzzy were weights. In this case, the fuzzy number used is a trapezoidal fuzzy number. In some cases, the search for the shortest route from the starting point to the destination point would be searched using the Artificial Bee Colony algorithm because this algorithm is very efficient and can be used with minimal control parameters, so to solve problems with fuzzy weights complete with membership functions a simple algorithm is needed, namely the Artificial Bee Colony algorithm. In numerical case examples, it is required to find the shortest route quickly and accurately, while in real cases in the field it is also required to find the shortest route and to minimize transportation time and costs.

This problem was then described in the form of a graph, where roads between cities or regions are depicted as lines (edges), while cities or regions are depicted as points (vertices). The concept of this ABC Algorithm was to be determined the initial solution which is then searched for another route using a local search until it finds the route with the smallest number of distances, the route with the smallest distance is the optimum solution. Based on this background, this research is entitled Artificial Bee Colony Algorithm to Solve the Shortest Route Problem with Fuzzy Arc Weight. This is in accordance with the words of Hasan Al-Basri (the great scholar from Iraq) in utilizing time effectively and efficiently, "O people, you are actually just a collection of days, every time a day is over, a part of you will also be lost.

Results and Discussion

During the long dry season in Gunung Kidul Regency, there was a severe water crisis, for this reason, sufficient and fast supply of clean water was needed for communities affected by the long dry season. The Banyu Water Company is one of the clean water companies in Gunung Kidul. The company received a request for clean water from the Karangmojo area, to reach the area there were several possible routes to take. Take a look at the route image below.

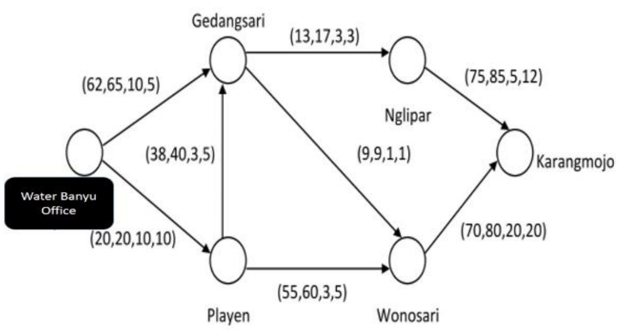

Figure 1. Case Example.

Node 1 – 2 = (20, 20, 10, 10)	Node 3 – 5 = (9, 9, 1, 1)
Node 1 – 3 = (62, 65, 10, 5)	Node 2 – 5 = (55, 60, 3, 5)
Node 2 – 3 = (38, 40, 3, 5)	Node 4 – 6 = (75, 85, 5, 12)
Node 3 – 4 = (13, 17, 3, 3)	Node 5 – 6 = (70, 80, 20, 20)

A trapezoidal fuzzy number L-R = (ḿ,m ,α,β) can be converted into an ordinary trapezoidal fuzzy number = (a_1,a_2,a_3,a_4). According to Wakas (2014), the way to change the L-R trapezoidal fuzzy number is very simple, as follows:

$$\tilde{A} = (\acute{m} - \alpha, \acute{m}, \grave{m}, \grave{m} + \beta) \quad (1)$$

So that the value of $a_1 = \acute{m} - \alpha$, $a_2 = \acute{m}$, $a_3 = \grave{m}$, $a_4 = \grave{m} + \beta$.

So that the fuzzy number is obtained as follows.
Node 1-2 = (20-10,20,20,20+10) = (10,20,20,30)
Node 1-3 = (52,62,65,70)
Node 2-3 = (35,38,40,45)
Node 3-4 = (10,13,17,20)
Node 3-5 = (8,9,9,10)
Node 2-5 = (52,55,60,65)
Node 4-6 = (70,75,85,97)
Node 5-6 = (50,70,80,100)

According to Kusumadewi (2003), limiting the number can speed up the calculation process, so that in this case the number of solutions is set equal to half of the total number of points, which is 3.

There are two parameters, namely the variable (D) and the number of candidate solutions (SN). Limit is the limitation of failed iterations in a calculation. The number of limits is the multiplication of the number of variables with the number of candidate solutions p(m)= (SN X D) (Nursyifa, 2013).

a) Step 1

Create an initial solution using Algorithm 1. Creating a solution or route from node 1 to node 6 using Algorithm 1. For i=1, m=1, and p(m)=1, we have a(1) = {2,3}. Then, randomly select the members of a_1 (1), for example j =2, and take m =2, and, since n 2, then i = 2 and a^1 (2)={3,5}. Now randomly select the members of a^1 (2), i.e. j=3,m=3, and p(m)=3, since n 3 then i=3 and a^1 (3)={4,5 }, then choose a random member of a^1 (3) i.e. j=4,m=4 and p(m)=4, because n 4 then i=4 and a^1 (4)=6 because n= 6 then the route is obtained as follows: (1,2,3,4,6).

Then in the same way finally we obtained the initial solution as follows.
1). 1, 2, 3, 4, 6
2). 1, 3, 4, 6
3). 1, 2, 5, 6

After that, the total distance was calculated using the -cut method with the following results.
Route 1 (1,2,3,4,6) = (125,146,162,192)
Route 2 (1,3,4,6) = (132, 150, 167, 187)
Route 3 (1,2,5,6) = (112,145,160,195)

b) Step 2

Employed bees make a local search around the solution using the mutation operator in the genetic algorithm (Hassanzadeh, 2013).

1) *New route from 1,2,3,4,6*

Choose a random number between 1 and 5, say 3. Then the components 1 to (3 – 1) are kept or fixed and the path starts from point 3 at position 3. To determine the next path, return to Algorithm 1. So that a^1 (2) = (3,5), then choose j=3,p(m)=(3) because n 6 then i=3,a^1 (3)=(4,5) , choose j=5,p(m)=(5), because n≠6 then i=5,a^1 (5)=6, because n = 6 so we get a new route from route 1,2,3,4,6

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based on the mutation operator as follows: 1,2,3,5,6.

So that the new route from the second and third routes are as follows:

2) *The new route of 1,3,4,6 is 1,3,5,6.*
3) *The route*

After obtaining a new route using the mutation operator, calculate the new route using the -cut method.

Route 1* (1,2,3,5,6) = (103,137,149,185)
Route 2* (1,3,5,6) = (110,141,154,180)
Route 3* (1,2,3,4,6) = (125,146,162,192)

After obtaining the new route, it was then compared with the old route.

According to Sub-chapter 2.2.8, to choose which path had the smaller value, the distance function $D_{2,\frac{1}{2}}$ was used.

Comparison of old route fuzzy numbers 1,2,3,4,6 from = (125, 146, 162, 192) with new route fuzzy numbers 1,2,3,5,6 from b = (103, 137, 149 , 185) with n = 10. Then the a_i cut of is as follows

$a^+_{a_i}$= (189, 186, 183, 180, 177, 174, 171, 168, 165, 162)

$a^-_{a_i}$= (146, 143.9, 141.8, 139.7, 137.6, 135.5, 133.4, 131.3, 129.2, 127.1)

For b was

$b^+_{a_i}$ = (181.4, 177.8, 174.2, 170.6, 167, 163.4, 159.8, 156.2, 152.6, 149)

$b^+_{a_i}$= (137, 133.6, 130.2, 126.8, 123.4, 120, 116.6, 113.2, 109.8, 106.4)

Based on the above results obtained

$$D_{2,\frac{1}{2}}(\tilde{\alpha}, MV) =$$
$$\sqrt{\left((189-0)2+\cdots\right)+\frac{1}{2}\left((146-0)2+\cdots\right)}$$
$$=$$
$$\sqrt{\left((35721+34596+\cdots)\right)+\frac{1}{2}(21316+20707.21+\cdots)}$$
$$= \sqrt{247783.925}$$
$$= 497.778992$$

In the same way, the comparison of the old route and the new route is obtained as follows.

Route 1 and Route 1*:
$(1,2,3,4,6) > (1,2,3,5,6)$
$497.778992 > 459.9142$

Route 2 and Route 2*:
$(1,3,4,6) > (1,3,5,6)$
$503.0422 > 467.6176$

Route 3 and Route 3*:
$(1,2,5,6) < (1,2,3,4,6)$
$489.9897 < 497.7778$

c) Step 3

The Onlooker Bee then chooses a path that depends on the probability associated with the fitness value of each path.

$$fit_i = \frac{1}{1+f_i}$$

Then the fitness value of each route was obtained as follows.

$$fit_1 = \frac{1}{1+459.9142} = 0.00216960119$$
$$fit_2 = \frac{1}{1+467.6176} = 0.00213393607$$
$$fit_3 = \frac{1}{1+489.9897} = 0.0020367026$$

Then the cumulative value is calculated.

$$\sum_{i=1}^{SN} fit_i = fit_1 + fit_2 + fit_3$$
$$= 0.00216960119 + 0.00213393607 + 0.0020367026$$
$$= 0.00634023986.$$

Furthermore, the cumulative value is used to calculate the probability value (pi) with the equation

$$p_i = \frac{fit_i}{\sum_{n=1}^{SN} fit_n}.$$
$$p_1 = \frac{0.00216960119}{0.00634023986} = 0.34219544337$$
$$p_2 = \frac{0.00213393607}{0.00634023986} = 0.33657024294$$
$$p_3 = \frac{0.0020367026}{0.00634023986} = 0.32123431368$$

The largest probability value obtained is p1 which was 0.34219544337, so the route that had the highest probability to be chosen by the Onlooker Bee was route 1 (1,2,3,5,6). After that, Onlooker Bee performs a local search on the selected route using the mutation operator and compares it with the previous route, so that routes 1,2,3,5,6 were obtained.

d) Step 4

In the scout bee phase, the cumulative probability value was compared with a random number between 0 to 1. The cumulative probability value is calculated by the equation

$$q_i = \sum_{k}^{i} p_k.$$

So that the cumulative value was obtained as follows.

$$q_1 = p_1 = 0.34219544337$$
$$q_2 = p_1 + p_2 = 0.34219544337 + 0.33657024294 = 0.67876568631$$
$$q_3 = p_1 + p_2 + p_3 = 0.34219544337 + 0.33657024294 + 0.32123431368 = 1$$

After that, a random number between 0 and 1.

$$r_1 = 0.9501$$
$$r_2 = 0.2311$$
$$r_3 = 0.6068$$

Then compared with the cumulative probability value.

$$q_1 < r_1$$
$$0.34219544337 < 0.9501$$
$$q_2 > r_2$$
$$0.67876568631 > 0.2311$$
$$q_3 > r_3$$
$$1 > 0.6068$$

e) Step 5

Based on the comparison results, the values of $q_2$and $q_3$are greater than random numbers, so that the route would be entered into the problem space and compared with the new route using algorithm 1.

Old route q_2 and new route q_2

(1, 3, 5, 6) > (1, 2, 3, 5, 6)

467.6176 > 459.9142

Old route $q_3$and new route q_3

(1, 2, 5, 6) > (1, 2, 3, 5, 6)

489.9897 > 459.9142

After this, iteration 1 was finished and goes to iteration 2, and so on until the number of failed iterations is equal to the number of the predetermined limit of 3

Iteration 1

No	Route	Interval Distance	Probability Value
1	1, 2, 3, 5, 6	459.9142	0.34219544337
2	1, 3, 5, 6	467.6176	0.33657024294
3	1, 2, 5, 6	489.9897	0.32123431368

Highest probability value: 0.344219544337

Best Routes: (1, 2, 3, 5, 6)

Best interval distance: 459.9142

Probability value used: 0.344219544337

Updated route:

No	Route	Distance
2	1,2,3,5,6	459.9142
3	1,2,3,5,6	459.9142

f) Step 6

The Artificial Bee Colony algorithm process was repeated and would stop when the termination criteria are met. Because until the last iteration there is no longer a route that had a better fitness than route 1,2,3,5,6, the shortest route obtained was route 1,2,3,5,6 with an interval distance of 459,9142 and the membership function was as follows

$$\mu_{1*}(x) = \begin{cases} \frac{(x-103)}{34}; 103 \leq x \leq 137 \\ 1; 137 \leq x \leq 149 \\ \frac{(185-x)}{36}; 149 \leq x \leq 185 \end{cases}$$

Conclusion

The application of the Artificial Bee Colony Algorithm in finding the shortest route on a network uses three operators, namely Scout Bee, Employed Bee, and Onlooker Bee. The method of finding the shortest route in a graph with the Artificial Bee Colony Algorithm where each line contains fuzzy numbers was almost the same as the steps in the application of the Artificial Bee Colony Algorithm in finding the shortest route on a network with real distances. The difference is in the sum of fuzzy numbers at each distance and how to compare one route to another. In this case, the sum of each line uses the α-cut sum and the route selection uses D_(2,1/2) and n = 10, so that the best route results are routes 1, 2, 3, 5, 6 with an interval distance value of 459 , 9142 and the membership function are as follows.

$$\mu_{1*}(x) = \begin{cases} \frac{(x-103)}{34}; 103 \leq x \leq 137 \\ 1; 137 \leq x \leq 149 \\ \frac{(185-x)}{36}; 149 \leq x \leq 185 \end{cases}$$

To get a supply of clean water, the Banyu Water Office must pass the Playen, Gedangsari, Wonosari routes, and go to the final destination, Karangmojo District.

The process of finding the shortest route with the Artificial Bee Colony Algorithm with fuzzy arc weight was still manual. For this reason, further researchers can make programs and develop programs to be faster and more efficient. In further research, the weight of the trapezoidal fuzzy number can be compared with other fuzzy numbers such as normal fuzzy numbers or triangular fuzzy numbers. This research is still limited to mathematical analysis, to find out whether the ABC Algorithm is effective or not, future research can use data from research results in the field. In future research, it is expected to be able to use and generate fuzzy numbers based on actual data from research results in the field so that the calculation results can be implemented directly in the field.

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