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Mathematics Model of Diabetes Mellitus Illness without Genetic Factors with Treatment

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Abstract

Diabetes Mellitus is a metabolic disorder characterized by an increase in glucose levels in the blood (hyperglycemia), which causes various chronic complications due to abnormalities in insulin secretion. Diabetes Mellitus is known as the Mother of Disease because it is the mother of various other diseases such as hypertension, heart disease, stroke and blindness. In this paper, what would be studied was a mathematical model of Diabetes Mellitus without genetic factors with treatment, the model used for the approach in this case was in the form of SEIIT. The analysis includes determining the model's equilibrium point, determining the basic reproduction number (R0) and analyzing the stability around the equilibrium point. Furthermore, numerical simulation using MAPLE was given based on the values of the related parameters in the mathematical model that describes the conditions in each subpopulation class.

***Keywords**: SEIIT, differential equation, equilibrium point, asymptotically stable.*

Introduction

Diabetes Mellitus represents a heterogeneous group of disorders (Boutayeb, at al., 2006). Some can be identified based on the cause or typical pathogenesis, but in many cases this process is not fully understood (Khalda'Aesa, 2020). Diabetes is characterized by hyperglycemia and disturbances of carbohydrate, fat and protein metabolism associated with insulin secretion (Purwoko, at al., 2019). Typical symptoms are excessive thirst (polydipsia), frequent urination (polyuria), frequent hunger (polypagia), pruritus, which is followed by fatigue, lack of energy and the body becomes thin despite eating a lot (Putra, at al., 2013).

In Indonesia, it is estimated that there are 3% – 4% of the total population or almost 6 million people with diabetes (Sutanegara & Budhiarta, 2000). In people with Diabetes Mellitus, it is usually due to the pancreas or stomach salivary glands being unable or insufficient to produce the insulin hormone that the body needs, so that the burning of carbohydrates as fuel for the body is less than perfect. This can lead to an increase in glucose (sugar) levels in the blood (Soewondo & Pramono, 2011). Glucose levels in the blood that are more than the normal limit, will be excreted through urine. The urine of a diabetic patient that has not been handled carefully will contain glucose (Ali sjahbana, at.al., 2006).

Diabetes Mellitus is classified into two main categories: Type 1 Insulin Dependent Diabetes Mellitus (IDDM) and Type 2 Non Insulin Dependent Diabetes Mellitus (NIDDM) (Tisch & McDevitt, 1996). Type 1 (IDDM) is a condition in which the body is unable to produce its own insulin, so insulin injections are needed (Atkinson & Maclaren, 1994). Type 2 diabetes (NIDDM) occurs because the beta cells of the pancreas do not produce enough insulin, causing the liver, muscle and fat do not react properly to the insulin produced in the minimum amount (Hales & Barker, 1992).

The hadith of the Prophet Muhammad that, narrated from Abu Huraira, Rasulullah said: "There is no disease that Allah has created, except Allah has created a cure". We were interested in studying the mathematical model of Diabetes Mellitus without genetic factors with treatments modeled in the form of $SEII_T$ (Brauer and Castillo-Chaves, 2011), (Driessche, 2002). The model would search for a disease-free equilibrium point and an endemic equilibrium point, the value of Basic Reproduction Number (R_0), then analyze the stability of the disease-free equilibrium point and the endemic equilibrium point and perform numerical simulations using MAPLE (Murray, J.D. 1993).

Results and Discussion

The assumptions used to form the mathematical model are as follows: the birth rate in the population is the same as the death rate, the effect of migration was

ignored that the spread of disease is closed in a population, there were no genetic factors that affect the spread of Diabetes Mellitus.

The positive parameters used are: the recruitment rate (birth rate) in the population is expressed by *A*. The natural death rate is expressed by μ. The infection contact rate of susceptible individuals to latent individuals is expressed by β. The rate of transfer of latent individuals to sick individuals without treatment is expressed by $\alpha\gamma$. The rate of transfer of latent individuals to sick individuals in the presence of treatment is expressed by $(1-\alpha\gamma)$. The death rate due to disease without treatment is expressed by δ_1. The rate of death due to disease in the presence of treatment is expressed by δ_2. Based on the assumptions formed, the appropriate model is the I_T model. To obtain a suitable model, the transfer diagram is given in Figure 1.

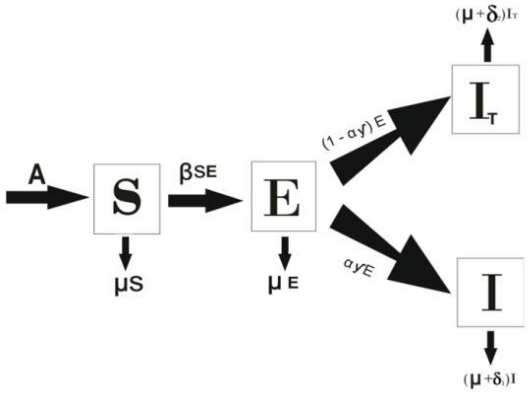

Figure 1. Transfer Diagram of Diabetes Mellitus without genetic factors with treatment.

Based on the picture of the mathematical model of Diabetes Mellitus without genetic factors with treatment consisting of four classes, namely the susceptible class of the susceptible population (*S*), the exposed class of the latent population (*E*), the ill class of the population infected with Diabetes Mellitus by receiving treatment (I_T) and the ill class population infected with Diabetes Mellitus who did not receive treatment (*I*).

$$\begin{aligned}\frac{dS}{dt} &= A-\mu S-\beta SE\\ \frac{dE}{dt} &= \beta SE-\mu E-E\\ \frac{dI}{dt} &= \alpha\gamma E-(\mu+\delta_1)I\\ \frac{dI_T}{dt} &= (1-\alpha\gamma)E-(\mu+\delta_2)I_T\end{aligned} \quad (1)$$

From System (1) obtained $\frac{dN}{dt}=\frac{dS}{dt}+\frac{dE}{dt}+\frac{dI}{dt}+\frac{dI_T}{dt}$. Because $I+I_T$, then $\frac{dS}{dt}=\frac{dN}{dt}-\frac{dE}{dt}-\frac{dI}{dt}-\frac{dI_T}{dt}$. So we get a simpler system, i.e

$$\begin{aligned}\frac{dN}{dt} &= A-\mu N-\delta_1 I-\delta_2 I_T\\ \frac{dE}{dt} &= \beta(N-E-I-I_T)E-\mu E-E\\ \frac{dI}{dt} &= \alpha\gamma E-(\mu+\delta_1)I\\ \frac{dI_T}{dt} &= (1-\alpha\gamma)E-(\mu+\delta_2)I_T\end{aligned} \quad (2)$$

The mathematical model of Diabetes Mellitus without genetic factors with treatment has two equilibrium points, namely the disease-free equilibrium point $P_0=(\frac{A}{\mu},0,0,0)$ and the endemic equilibrium point $P_1=(N^*,E^*,I^*,I_T^*)$, where

$$\begin{aligned}N^* &= \frac{A}{\mu}-\frac{\delta_1\alpha\gamma\mu(\beta A-\mu-\mu^2)}{(\mu+\delta_1)\beta(1+\mu)}\\ &\quad -\frac{\delta_2(1-\alpha\gamma)\mu(\beta A-\mu-\mu^2)}{(\mu+\delta_2)\beta(1+\mu)}\\ E^* &= \frac{A\beta-\mu(\mu+1)}{\beta(1+\mu)}\\ I^* &= \frac{\alpha\gamma(\beta A-\mu-\mu^2)}{\beta(\mu+\mu^2+\delta_1+\mu\delta_1)}\\ I_T^* &= \frac{(1-\alpha\gamma)(\beta A-\mu-\mu^2)}{\beta(\mu+\mu^2+\delta_2+\mu\delta_2)}.\end{aligned}$$

The endemic equilibrium point P_1 exists, if $R_0>1$. The basic reproduction number (R_0) in this disease model was

$$R_0=\frac{\beta A}{\mu(\mu+1)}.$$

Furthermore, the stability will be analyzed at each equilibrium point. The Jacobian matrix for the mathematical model of Diabetes Mellitus without genetic factors with treatment was

$$J_f(x)=\begin{bmatrix}-\mu & 0 & -\delta_1 & -\delta_2\\ \beta E & \beta(N-2E-I-I_T)-\mu-1 & -\beta E & -\beta E\\ 0 & \alpha\gamma & -(\mu+\delta_1) & 0\\ 0 & 1-\delta_1 & 0 & -(\mu+\delta_2)\end{bmatrix}$$

For diseases $P_0=(\frac{A}{\mu},0,0,0)$ and $P_1=(N^*,E^*,I^*,I_T^*)$, where

$N^*=\frac{A}{\mu}-\frac{\delta_1\alpha\gamma\mu(\beta A-\mu-\mu^2)}{(\mu+\delta_1)\beta(1+\mu)}-\frac{\delta_2(1-\alpha\gamma)\mu(\beta A-\mu-\mu^2)}{(\mu+\delta_2)\beta(1+\mu)}$,

$E^*=\frac{A\beta-\mu(\mu+1)}{\beta(1+\mu)}$,

$I^*=\frac{\alpha\gamma(\beta A-\mu-\mu^2)}{\beta(\mu+\mu^2+\delta_1+\mu\delta_1)}$,

$I_T^*=\frac{(1-\alpha\gamma)(\beta A-\mu-\mu^2)}{\beta(\mu+\mu^2+\delta_2+\mu\delta_2)}$.

For the case of P_1 obtained a negative eigenvalue for $R_0>1$ so that the endemic equilibrium point of P_1 is locally asymptotically stable. While for P_0 obtained

eigenvalues $\lambda_1 = -\mu$, $\lambda_2 = -\mu - \delta_1$, $\lambda_3 = -\mu - \delta_2$ and $\lambda_4 = \beta\frac{A}{\mu} - 1 - \mu$. Because all the parameters used are positive, then for $R_0 < 1$ point P_0 is local asymptotically stable and if $R_0 > 1$ then P_0 is unstable.

Simulation was done by assigning values to each parameter according to R_0. This simulation was given to provide a geometric picture of the existence and stability theorem of the equilibrium points of this model.

Based on the explanation of the meaning of the parameter values, the value of *A* was the recruitment rate (birth rate) in the population, the value of μ the natural death rate, the value of β the rate of infective contact of susceptible individuals to latent individuals, the value of $\alpha\gamma$ the rate of transfer of latent individuals to sick individuals without treatment, the value $(1-\alpha\gamma)$ of the rate of transfer of latent individuals to sick individuals with the treatment, the value of δ_1 the rate of death from disease without treatment and the value of δ_2 the rate of death from disease with the treatment. It is assumed that the individual death rate was 63 years, then

$$\mu = \frac{1}{63\times12} = 0.00132,\ \beta = 0.0009$$

This means that there are 9 susceptible individuals who become latent if there were 1000 susceptible individuals who come into contact with latent individuals, it was assumed that the rate of transfer of latent individuals to become ill was 20 years, the death rate due to diabetes was 60 years, the death rate due to diabetes with the effect of care is 62 years, so that obtained

$$\alpha\gamma = \frac{1}{20\times12} = 0.004,$$
$$\delta_1 = \frac{1}{60\times12} = 0.00139,$$
$$\delta_2 = \frac{1}{62\times12} = 0.00134.$$

A. Simulation for $R_0 < 1$

Simulation for $R_0 < 1$, given the parameter values, i.e.

Table 1. Parameter values for $R_0 < 1$.

Parameter	Value	Parameter	Value
A	1	δ_2	0.00134
μ	0.00132	β	0.0009
δ_1	0.00139	$\alpha\gamma$	0.004

From the parameter values that had been given, the value of $R_0 = 0.68$ was obtained. So that one equilibrium point is obtained when $R_0 < 1$, namely the point $P_0 = (\frac{A}{\mu}, 0,0,0)$ locally asymptotically stable. To illustrate the conditions in class *N*(*t*), *E*(*t*), *I*(*t*) and $I_T(t)$ graphs were presented in Figure 2 and Figure 3.

In Figure 2 and Figure 3, respectively, $N(t) \longrightarrow \frac{A}{\mu}$ means that with increasing time the number of individuals in the vulnerable population will reach the equilibrium point of *N*(*t*). $E(t) \longrightarrow 0$, $I(t) \longrightarrow 0$, and $I_T(t) \longrightarrow 0$ means that the number of individuals in each population will decrease over time.

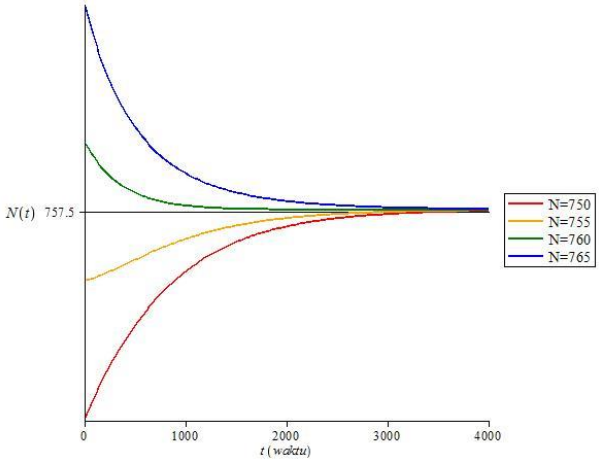

Figure 2. Phase portrait *N*(*t*) when $R_0 < 1$.

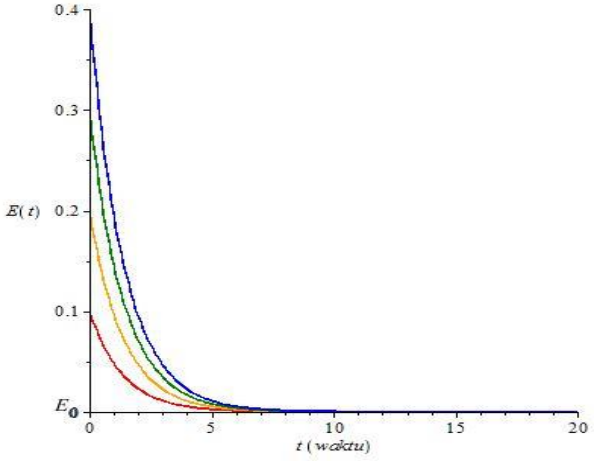

Figure 3. Phase portrait *E*(*t*)) when $R_0 < 1$

B. Simulation for $R_0 > 1$

Simulation for $R_0 > 1$, given the parameter values, i.e.

Table 2. Parameter values for $R_0 > 1$.

Parameter	Value	Parameter	Value
A	2	δ_2	0.00134
μ	0.00132	β	0.0009
δ_1	0.00139	$\alpha\gamma$	0.004

From the parameter values that had been given, the value $R_0 = 1.36$ was obtained. So that we get 2 equilibrium points when $R_0 > 1$, namely point $P_0 = (\frac{A}{\mu}, 0,0,0)$ unstable and $P_1 = (N^*, E^*, I^*, I_T^*)$ asymptotically stable local. To illustrate the conditions in classes *N*(*t*), *E*(*t*), *I*(*t*) and $I_T(t)$ graphs are presented in Figure 4, Figure 5, Figure 6, and Figure 7.

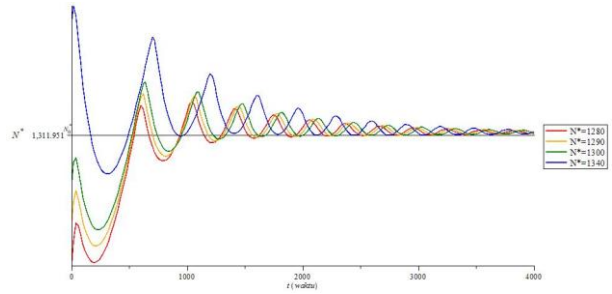

Figure 4. Phase portrait $N^*(t)$ when $\mathrm{R}_0 > 1$.

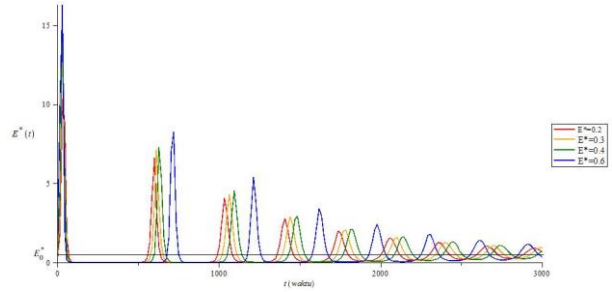

Figure 5. Phase portrait $E^*(t)$ when $\mathrm{R}_0 > 1$.

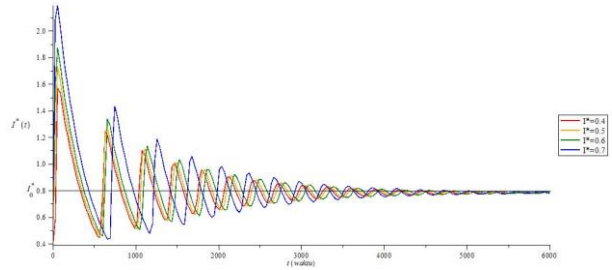

Figure 6. Phase portrait $I^*(t)$ when $\mathrm{R}_0 > 1$.

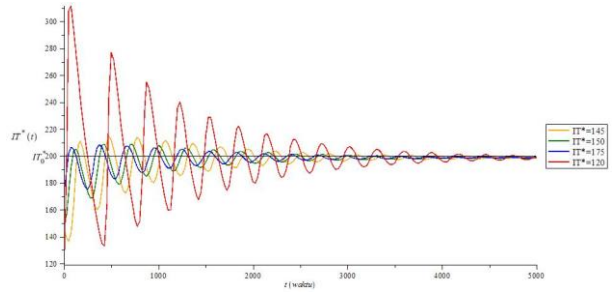

Figure 7. Phase portrait $I^*(t)$ when $\mathrm{R}_0 > 1$.

From Figure 4 obtained

$$N^*(t) \longrightarrow \frac{A}{\mu} - \frac{\delta_1 \alpha\gamma\mu(\beta A - \mu - \mu^2)}{(\mu+\delta_1)\beta(1+\mu)} - \frac{\delta_2(1-\alpha\gamma)\mu(\beta A - \mu - \mu^2)}{(\mu+\delta_2)\beta(1+\mu)}$$

It means that with increasing time the number of individuals in the vulnerable population would go to the equilibrium point $N_0^*(t)$. From Figure 5, it was obtained that $E^*(t) \longrightarrow E_0^*(t) = 0.5$ means that the number of latent individuals would always be present in the population due to infective contact for subpopulations *S*(*t*) and *E*(*t*). From Figure 6, it is obtained that $I^*(t) \longrightarrow I_0^*(t) = 0.8$ means that the number of sick individuals will always exist in the population because there are still individuals who have bad habits. From Figure 7, it was obtained that $I_{T_0}^*(t) \longrightarrow I_{T_0}^*(t) = 200$ means that the number of individuals who were sick with treatment will always exist in the population because there are still individuals who had bad habits.

Conclusion

From the above presentation, it was obtained

1. The value of Basic reproduction number (R_0) for the model of Diabetes Mellitus without genetic factors with treatment was $R_0 = \frac{\beta A}{\mu(\mu+1)}$.
2. The mathematical model of Diabetes Mellitus without genetic factors with treatment had two equilibrium points, namely:
   a. Disease-free equilibrium point $P_0 = (\frac{A}{\mu}, 0,0,0)$..
   b. Titik ekuilibriu mendemik $P_1 = (N^*, E^*, I^*, I_T^*)$ dengan,
   c. The equilibrium point is $P_1 = (N^*, E^*, I^*, I_T^*)$ with,
   $N^* = \frac{A}{\mu} - \frac{\delta_1 \alpha\gamma\mu(\beta A-\mu-\mu^2)}{(\mu+\delta_1)\beta(1+\mu)} - \frac{\delta_2(1-\alpha\gamma)\mu(\beta A-\mu-\mu^2)}{(\mu+\delta_2)\beta(1+\mu)}$,
   $E^* = \frac{A\beta-\mu(\mu+1)}{\beta(1+\mu)}$,
   $I^* = \frac{\alpha\gamma(\beta A-\mu-\mu^2)}{\beta(\mu+\mu^2+\delta_1+\mu\delta_1)}$
   and $I_T^* = \frac{(1-\alpha\gamma)(\beta A-\mu-\mu^2)}{\beta(\mu+\mu^2+\delta_2+\mu\delta_2)}$.
3. The disease-free equilibrium point $P_0 = (\frac{A}{\mu}, 0,0,0)$ was locally asymptotically stable with the condition that $\mathrm{R}_0 < 1$, and the endemic equilibrium point $P_1 = (N^*, E^*, I^*, I_T^*)$ local asymptotically stable with condition $\mathrm{R}_0 > 1$.

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