

## Exploring Students' Visual Thinking: Examining Students' Sequence And Series Analysis Through The Lens Of Visual Cognitive Styles

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### ABSTRACT

Visual thinking skill has an important role in solving mathematics problems. Visual thinking not only can be applied on the topic of geometry but also on the other topics including sequence and series. The visual cognitive styles have a relationship to mathematical abilities and types of tasks. This research aims to analyze the students' visual thinking skill viewed by visual cognitive styles on the topic of sequence and series used grounded theory approach. The steps in this study are developing visual cognitive style questionnaire and visual thinking skill test, asking students to fill the questionnaire and do the test, and analyzing the results of questionnaires and students' answer. The subjects of this research were 2 students at the eleventh grade (16-17 year-olds). The result showed that students with spatial visualization have better visual thinking skill than object visualization. Students with spatial visualization are very good at working on problems on indicators transforming problems or concepts into visual forms. Whereas, students with object visualization are very good at drawing for the  $n$ th term of a sequence. The analysis of this research can be used as material for consideration to make the learning design on the topic of sequence and series that is in accordance with the cognitive styles of students.

**Keywords:** Visual Thinking, Visual Cognitive, Sequence and Series

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### INTRODUCTION

Students are expected to master visual thinking skills in solving mathematical problems (Juandi & Priatna, 2018). Visual thinking has an important role in the development of thinking, mathematical understanding, and the transition from abstract to concrete related to solving mathematical problems. Arcavi defines visual thinking as the ability, the process, and the product of creation, interpretation, use of, and reflection upon images, diagrams, in our minds, on paper or with the help of technology, with the aim of describing and communicating information, thinking about and developing ideas that are not previously known, and advancing understandings (Arcavi,

2003). Therefore, visual thinking ability can be interpreted as a person's ability to understand and interpret information that involves relevant images, process images and transform ideas into visual forms.

The visual thinking skill is interesting to be discussed because the results of the study found that students experience limitations and difficulties in building a visual representation. Students' difficulties are understanding problems, drawing diagrams, reading charts correctly, understanding conceptual formal mathematics, and mathematical problem solving (Arcavi, 2003; Eisenberg & Dreyfus, 1991; Stylianou & Silver, 2004). Students can understand the problem better when they can provide visual images that represent the situation in the mathematical problems they face. The visual thinking skill can help students in stating the mathematical problems with their own language.

Each individual has its own characteristics that distinguish it from other individuals. Each individual has their own way of receiving and processing information when solving problems. This is what is known as cognitive style. The cognitive style is a typical way that is consistent in receiving and processing information to be used in solving problems. Cognitive style related to the habits of students using sensory devices is divided into two, namely verbal and visual cognitive styles (McEwan & Reynolds, 2007). The cognitive visual style consists of spatial visualization and object visualization. The cognitive visual style has a relationship with mathematical abilities and certain types of tasks.

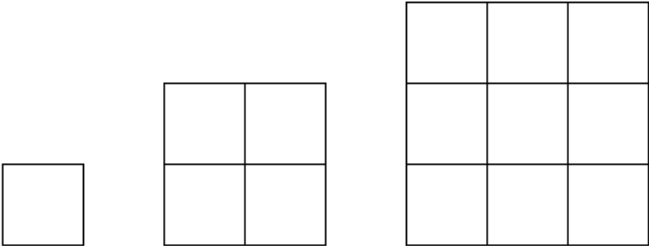
Visual thinking skill can have a relationship with other mathematical abilities and the differences in the types of visual cognitive styles has an effect on different topics of mathematics and tasks (Huang, 2015; Pitta-Pantazi, et.al., 2013). Therefore, it is important to identify students included in spatial visualization or object visualization. Identification of the time of use and visual images used can facilitate students in understanding mathematics. This is useful for teachers to design learning that is more suitable for each student. The researchers about visual thinking skills often used geometry subject (Pitta-Pantazi, et.al., 2013; Pitta-Pantazi & Christou, 2010). Even though, visual thinking skill can also be used in others subject. Expert in mathematics used visual thinking skills to solve not only in geometry subject but also in various problems in mathematics (Stylianou & Silver, 2004).

Taking those described problems into account, this research aims to analyse students' visual thinking viewed by visual cognitive styles on the topic of sequence and series. The result of this study can be used as supporting information to develop further learning design.

## METHOD

This research is qualitative method with a grounded theory approach. The stages of grounded theory are open coding, axial coding, and selective coding. The subjects of this research were 2 students at the eleventh grade (16-17 year-olds). The students came from a public school in Bandung. This research aims to analyse students' visual thinking viewed by visual cognitive styles on the topic of sequence and series. To do so, first, we developed visual cognitive styles questionnaire and visual thinking skill test on the topic of sequence and series. We used Object-Spatial Imagery Questionnaire (OSIQ) that contain 30 statements (Blajenkova, et.al., 2006). The visual thinking skill test contain three tasks based on the indicators of visual thinking skill (see Table 1). Next, we requested students to fill the questionnaire in 20 minutes and solve the test in 60 minutes. Then, we analysed students' answer.

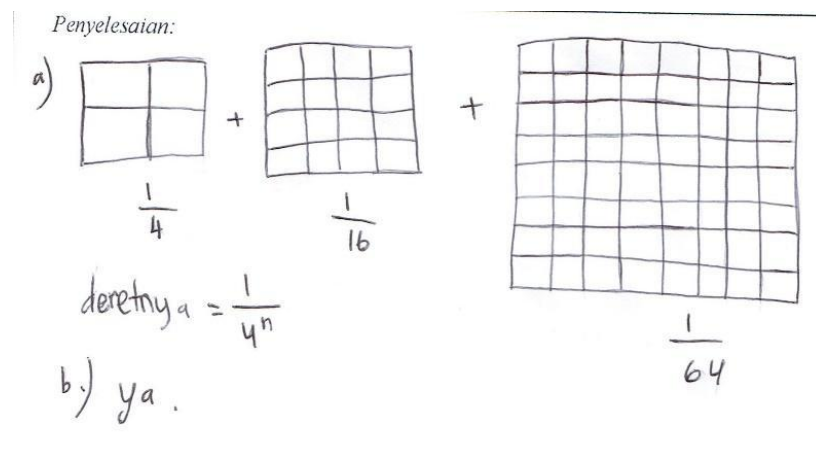
**Table 1.** Visual Thinking Skill Test

No	Indicator	Task
1.	Transform the problem or concept into visual form	<p>Note this following series.</p> $\frac{1}{4} + \frac{1}{16} + \frac{1}{64} + \dots = \frac{1}{3}$ <p>a. Make a image that represent the series. b. Can you see the result of the series from the images?</p>
2.	Obtain specific information from images	<p>Note the following images.</p> <div style="text-align: center;">  </div> <p>a. Compose a sequence that represent the number of the squares in the images. b. Find the 5<sup>th</sup> term of the sequence.</p>
3.	Construct the images as help in solving problems	<p>A ball has a bounce power of 65% as high as the preceding height. If the ball is dropped from a height of 5 m, find the height does it reach after 3th bounce.</p>

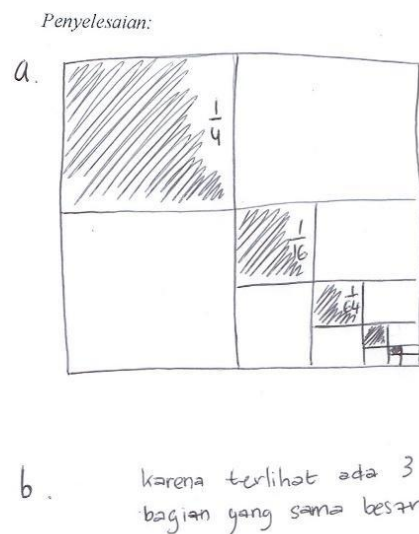
## RESULTS AND DISCUSSION

In this research, there are 2 students with spatial visualisation and object visualisation. This classification of cognitive styles is based on the results of the visual cognitive style questionnaire. The students were given three tasks about sequence and series based on visual thinking skill indicators. For Task 1, student with object visualisation use squares to represent the series (see Figure 1). He divided the square into 4 equal parts to represent 1/4. Then, he made a square again and divided it into 16 part to represent 1/16 and so on. He get that the series is  $1/4^n$ . But, we cannot see the result of the series is equal to 1/3 from the figure that he created to represent the

series. This indicates that student with object visualisation see a problem part by part. This is in line with the findings of Kozhevnikov which stated that students with object visualisation use imagery to construct high-quality images of the shapes of individual objects (Kozhevnikov, et.al., 2005).



**Figure 1.** The answer of student with object visualisation in Task 1



**Figure 2.** The answer of student with spatial visualisation in Task 1

Figure 2 showed the answer of student with spatial visualisation in Task 1. He used a square to construct the representation of the series. First, he divided a square into 4 equal parts to represent  $1/4$ . Then, he divided the part that represent  $1/4$  into 4 equal parts to represent  $1/16$  and so on. We can see that the shaded part is represent the series. The shaded part is equal with the others. We can see from the figure there are 3 equal parts. So, we can see the result of the series from the figure made by him. This means he understands the relation between the images and the series. This implies that student with spatial visualisation see a problem thoroughly. Student with spatial

visualisation create the visual representation of the series, so that the visual representation can give important information to solve the problem (Hegarty & Kozhevnikov, 1999). This is related to the research of Kozhevnikov dkk. found that students with spatial visualisation use imagery to represent and transform spatial relations (Kozhevnikov, et.al., 2005).

In Task 2, it is found that student with object visualisation cannot determine all the square in the images (see Figure 3). He only seen 4 squares on the second term and 9 squares on the third term. While, student with spatial visualisation can see all the square in the images correctly (see Figure 4). This implies that student with object visualisation tend to see images globally as a single perceptual unit. In contrast, student with spatial visualisation tend to see images part by part. Students with object visualisation process images holistically, while students with spatial visualisation encode and process images analytically, using spatial relations to arrange and analyze the components (Kozhevnikov, et.al., 2005).

Task 2 asked to find 5th term of an arithmetic sequence. Both of student with spatial and object visualisation do not use the formula for a general term for an arithmetic sequence. They used the concept of the sequence and series. Sequence is a function that has a pattern. So, they found the pattern to solve the problem. Student with object visualisation can see the pattern of the sequence from the first three term. He noticed that  $U_n = n^2$ . While, student with spatial visualisation determined the common difference of the sequence. Then, he notice that the common difference is quadratic. So, he found that  $U_n = U_{(n-1)} + n^2$ . This implies that students with visual cognitive style have a relation with creativity because they solved the problem using their own way. It seems that this is caused by the questions only asking to find the term that is not too large so students feel it is easier not to use the formula in solving it. This cause them to be more creative and provide different solutions to solving problems (Pitta-Pantazi, et.al., 2013; Pitta-Pantazi & Christou, 2010).

Penyelesaian:

a.) 1, 4, 9, ...

b.)  $U_n = n^2$   
 $= 5^2$   
 $= 25 //$

**Figure 3.** The answer of student with object visualisation in Task 2

Penyelesaian:

+4 +9  
 1, 5, 14, ...  
 $U_1 \quad U_2 \quad U_3$

Suku ke 4 =  $U_4 = U_3 + n^2 = 14 + 4^2 = 14 + 16 = 30$   
 Suku ke 5 =  $U_5 = U_4 + n^2 = 30 + 25 = 55$

$U_n = U_{n-1} + n^2$

suku ke 5

**Figure 4.** The answer of student with spatial visualisation in Task 2

In Task 3, student with object visualisation used the formula for a general term for a geometric sequence to solve the problem (see Figure 5). He looked for the third term of the sequence to answer the task. But, the height of the ball after third bounce actually is fourth term of geometric sequence with the ratio 0.65 and the first term is 5. He made mistake because he did not construct the image of ball bouncing to make it easier to solve the task. In contrast, student with spatial visualisation made the image of ball bouncing. So, he knew what the task asks and what the solution needed to solve the task.

Student with spatial visualisation did not use the formula for a general term for a geometric sequence to solve Task 3 (see Figure 6). He knew that the height of the ball after bounce is 65% as high as the preceding height. So, he noticed that the height of the ball after bounce is  $U_n = 65\% \times U_{(n-1)}$ . We called this as recursive formula. Recursive formula is a formula that uses the previous term to get to the next one. This means student with spatial visualisation uses visual

representation as an aid to solve the task. It is easier to solve the task if we use visual representation and we can avoid doing mistakes. In line with that, Stylianou dan Silver stated that using visual representation is a viable strategy for problem solving because it can facilitate the problem solving process. Student with spatial visualisation solve the task in a different way (Stylianou & Silver, 2004). The result of a research also found similar with this findings. One of the characteristics of students with spatial visualisation is creative (Kozhevnikov, et.al., 2005).

Penyelesaian:

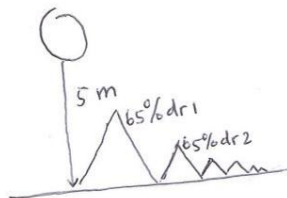
$$r = \frac{65\%}{100\%} = \frac{13}{20}$$

$$a = 5$$

$$\begin{aligned} u_3 &= ar^{n-1} \\ &= 5 \cdot \frac{13}{20}^{3-1} \\ &= 5 \cdot \left(\frac{13}{20}\right)^2 \\ &= 5 \cdot \frac{169}{400} \\ &= 2,1125 \end{aligned}$$

**Figure 5.** The answer of student with object visualisation in Task 3

Penyelesaian:



dr tingginya 5 m

$$U_1 = 65\% \times 5 \text{ m} = 3,25$$

$$U_2 = 65\% \times 3,25 = 2,1125$$

$$U_3 = 65\% \times 2,1125 = 1,373125$$

$$U_n = 65\% \times (U_{n-1})$$

**Figure 6.** The answer of student with spatial visualisation in Task 3

Based on the students' answer when solving visual thinking skill test, we found that student with spatial visualisation has better visual thinking skill in solving sequence and series task.

Student with spatial visualisation really good at making an image to represent the series. Student with object visualisation made visual representation separately, according to the terms. This indicates that student with object visualisation see a problem part by part. Meanwhile, the student with spatial visualisation made visual representation of a series in an image. So, they think and see a problem thoroughly.

In case of obtaining specific information from an image, object visualizers tend to encode images globally as a single perceptual unit, which they process holistically. So, they can miss the information that display implicitly in the images. On the other side, spatial visualizers tend to encode and process images analytically, part by part, using spatial relations to arrange and analyse the components. This means students with spatial visualisation can get whole information from the images because they are thorough when process the images.

When solving the sequence and series tasks, student with visual cognitive style likely did not use the formula of sequence and series. Student with object visualisation only use formula for a general term for a geometric sequence. This maybe due to the question just asks for the lower terms. So, students think it is easier to solve the tasks without using the formula. This indicates that student can provide different strategy to solve the task. It also implies that visual cognitive styles might have relationship with creative skill.

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## **CONCLUSION**

The research question addressed in this paper concerns about students' visual thinking skill viewed by visual cognitive styles on the topic of sequence and series. The results lead to the following conclusions. Student with spatial visualisation has better visual thinking skill in solving sequence and series task. Student with object visualisation made visual representation separately, according to the terms. Meanwhile, the student with spatial visualisation made visual representation of a series in an image. Object visualizers tend to encode images globally as a single perceptual unit. In contrast, spatial visualizers tend to encode and process images analytically, part by part. Student with visual cognitive style likely did not use the formula of sequence and series when solving the sequence and series tasks.



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